1026-1265/99

Physically-based Adaptive Control of Cavity Pressure in Injection Moulding Process: Packing Phase

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Received: 11 April 1998; accepted 26 January 1999

ABSTRACT

Cavity pressure plays an important role in determining the quality of injection moulded article. However, the non-linear and time varying behaviour of the process causes difficulties in implementing efficient control systems, since these generally employ linear time invariant design strategies. Conversely, advanced control methods, such as self-tuning, do not provide physical information about the process. A physically-based adaptive controller has been developed for cavity pressure control during the packing phase. Experimental results indicate that this control yields accurate results for the injection moulding of polyethylene. Different control algorithms with different tuning criteria, were implemented on a microcomputer to control the cavity pressure. The results also show that this method is superior to the classical empirical control approach.

Key Words: injection moulding, packing phase, adaptive control, physically-based, modelling

INTRODUCTION

The injection moulding process for thermoplastic materials is an important and widely used process for low cost products. Based on cavity pressure, each injection moulding cycle can be divided into three phases: filling, packing and cooling. Consistency and repeatability from cycle to cycle are important objectives of process control. Hence, some key variables should be controlled. It has been shown that cavity pressure is the variable of choice for achieving control of the injection moulding cycle [1]. The dynamic, cyclic and time varying nature of this process renders its control very challenging. Allen [2] discussed the importance of including process variables in control and urged the control of melt properties rather than machine variables. Keyes [3] studied the advantages and disadvantages of different variables and concluded cavity pressure was one of the most significant variables to control. Sanschagrin [4] determined the interaction between ten process inputs and three outputs using a closed-loop control system. Agrawal et al. [5] reviewed various control strategies. They categorized control strategies into all-phase,

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phase dependent, and cycle-to-cycle controls. So far, process identification techniques have been applied to the injection moulding process to develop dynamic models for the various parameters of the process.

Kamal et al. [6] carried out a detailed study on dynamics and control of hydraulic, nozzle, and cavity pressures. They obtained their model using steps and pseudo random binary sequences (PRBS) test on the machine. Dong and Tseng [7] discussed the stochastic dynamic characteristics of the process and designed a multi-variable self-tuning controller. They showed the effectiveness of the controller by computer simulation. Smud et al. [8] applied some advanced model-based algorithms such as simplified model predictive control (SMPC) and conservative model based control (CMBC) on the process. They used a deterministic first order plus dead time (FOPDT) model. Patterson et al. [9, 10] designed a self-tuning controller for cavity pressure control. Their controller showed very good control of cavity pressure.

The experimental models have been used successfully for control purposes, but these models are black box which relates input to output without giving much insight into the process. Although there have been some attempts, but theoretical models have not been used in controller design. Shankar and Paul [11] presented a non-linear lumped model for the process. They found some model parameters experimentally. They did not present any control strategy.

Hu and Vogel [12] used a modelling approach similar to that developed by Shankar. They designed a proportional integral derivative (PID) controller. However, the experimental results showed a large overshoot that was attributed to the linearization of model used in controller design. Recently, Rafizadeh et al. [13] modelled the process using same approach.

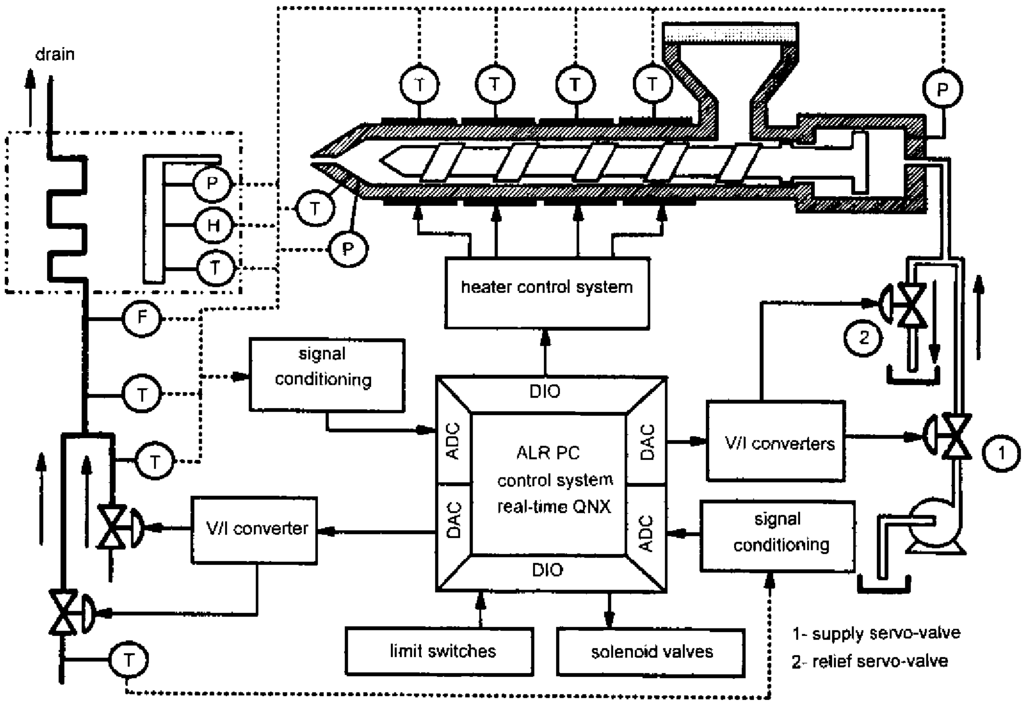

Figure 1. McGill Injection moulding machine.

They considered the whole process and took into account the elasticity behaviour of polymer melt and solidification of melt in the polymer delivery system. Their results were in good agreement with experimental data.

The non-linearity and time varying nature of the process suggest the use of an adaptive controller. This paper presents an adaptive control strategy using a physically-based model of the process [13] for the packing phase.

EXPERIMENTAL

Experimental work is carried out on a 60 tons Danson-Metalmec 2 1/3 oz, reciprocating screw injection moulding machine. The hydraulic system had been modified to incorporate two servo-valves to have much more effective control ability. Figure 1 shows the schematic of the McGill injection moulding machine. The two servo-valves are numbered 1 and 2 for supply and relief servo-valves, respectively. The percent openings of the servo-valves are related such as [14]:

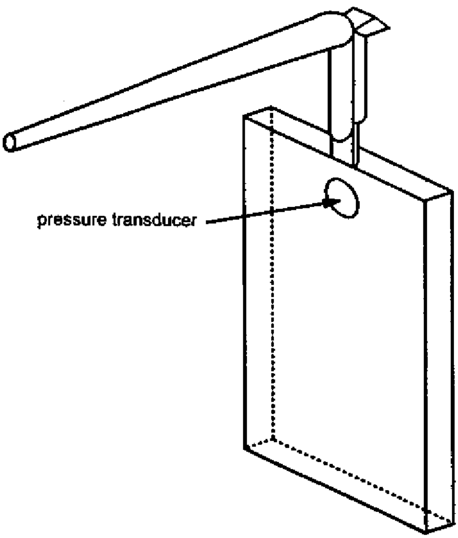

Figure 2. Mould cavity.

$$i_R = 0.2 \times (100 - i_s) + 0.4 \tag{1}$$

where i_s and i_R are the percent opening of the supply and relief servo-valves, respectively.

A rectangular cavity, equipped with a pressure transducer near the gate, is used. Figure 2 shows the cavity and location of the pressure transducer. The four barrel temperature zones are set at 205, 195, 180, 150 °C, respectively, unless stated otherwise.

Data acquisition and monitoring and control are done with a personal computer. The software for above purposes is developed internally. A sampling period of 10 millisecond is determined to be sufficient for control purposes.

An injection moulding grade high density polyethylene (Sclair 2908), supplied by Dupont Canada, is used. Complete physical and rheological properties of the resin have been reported elsewhere [15].

THE NON-LINEAR STATE EQUATIONS

An effective control system is dependent on understanding the process dynamics. The following set of non-linear state equations governs the polymer melt and hydraulic oil during the packing phase:

$$\frac{dq_r}{dt} = F_1 = -\frac{1}{\tau} q_r + \frac{k_v \sqrt{P_h}}{\tau} i_R \tag{2a}$$

$$\frac{dq_s}{dt} = F_2 = -\frac{1}{\tau} q_s + \frac{k_v \sqrt{P_s - P_h}}{\tau} i_s \tag{2b}$$

$$\frac{dP_h}{dt} = F_3 = \frac{\beta_h}{V_{ho} + A_h z}(q_s - q_r - A_h v_z) \tag{2c}$$

$$\frac{dz}{dt} = F_4 = v_z \tag{2d}$$

$$\frac{dv_z}{dt} = F_5 = \frac{1}{M}[A_h P_h - A_n P_n - 2\pi \eta_0 R_n^{1-n} (l_0 + z) \frac{(s-1)^n}{(k_r^{1-s} - 1)^n} v_z^n] \tag{2e}$$

$$\frac{dP_n}{dt} = F_6 = \frac{\beta_p}{V_{n0} - A_n z}[A_n v_z - n\sqrt{\frac{P_n - P_{gate}}{k_t}}] \qquad (2f)$$

$$\frac{dP_{gate}}{dt} = F_7 = \frac{\beta_p^n}{V_t}\sqrt{\frac{P_n - P_{gate}}{k_t}} \qquad (2g)$$

This set of governing equations is based on the laws of conservation principles and includes basic information of process physics. The two first equations concern with the servo-valves dynamics. The third and sixth and seventh equations are application of integral form of mass balance on the injection cylinder, nozzle and cavity, respectively. It is assumed that pressure is uniform throughout the injection cylinder, nozzle, and cavity. The fifth equation is derived by Newton's second law applied to the screw. The friction force is approximated by the axial annular Couette flow. The polymer melt treated as the shear thinning power law fluid, k_t, in the sixth and seventh equations, is the total resistance against flow between the nozzle and the cavity mould. Hagen-Poiseuille for the power law fluid is used for the runner. For sprue, which has a very small diverging angle, the fully-developed flow assumption is used. The normal stress effect is taken into consideration in converging zone of the nozzle and radial part of the cavity mould.

Linearized Model

Although much work has been done in the area to develop non-linear control theory for system design, much of the theory is applicable to simple low order systems. For the case considered here, the linearized model is used. The linearized set of equations (in Laplace transformed form) is:

$$sQ_r(s) = -\frac{1}{\tau}Q_r(s) + \frac{a}{\tau}H(s) + \frac{b}{\tau}R(s) \qquad (3a)$$

$$sQ_s(s) = -\frac{1}{\tau}Q_s(s) + \frac{c}{\tau}H(s) + \frac{d}{\tau}T(s) \qquad (3b)$$

$$sH(s) = eQ_r(s) - eQ_s(s) - fZ(s) - A_h eV(s) \qquad (3c)$$

$$sZ(s) = V(s) \qquad (3d)$$

$$sV(s) = gH(s) - iZ(s) - jV(s) - kN(s) \qquad (3e)$$

$$sN(s) = lZ(s) - mV(s) - nN(s) - nP_c(s) \qquad (3f)$$

$$sP_c(s) = pN(s) - pP_c(s) \qquad (3g)$$

Appendix A gives the related parameters. The algebraic manipulation reveals the transfer function which relates the controlled variable (the cavity pressure at the gate) to the manipulated variable (supply servo-valve opening) which is:

$$P_c(s) = \frac{pge(l + ms)}{DEN.}(d + 0.2b)T(s)$$
$$= \frac{{}^{*}n_1 s + n_2}{\tau(s - r_1)(s - r_2)(s - r_3)(s - r_4)(s - r_5)(s - r_6)}T(s) \qquad (4a)$$

$$DEN. = \tau s^6 + (\tau n + \tau p + \tau j + l)s^5 +$$
$$[\tau i + \tau jn + \tau jp + \tau km + n + p + j + \tau g A_h e + e(c + a)]s^4$$
$$+ [\tau in + \tau ip + \tau kl + \tau kmp + jn + jp + i + km + \tau gf$$
$$+ \tau g A_h en + \tau g A_h ep + g A_h e + (en + ep + ej)(c + a)]s^3$$
$$+ [klp\tau + in + ip + kl + kmp + \tau gnf + \tau gpf + fg$$
$$+ g A_h en + g A_h ep + (ejn + ejp + ei + ekm)(c + a)]s^2$$
$$+ [klp + gnf + gpf + (ein + eip + ekl + ekmp)(c + a)]s$$
$$+ eklp(c + a) \qquad (4b)$$

This is representation of a linear and time-

Table 1. Roots of transfer function, i_s=40%, i_R=15% and t=6.6 s.

Zero and poles	Values
Zero	-5.539×10^{-5}
First pole	$-42.34+239.26\times i$
Second pole	$-42.34-239.26\times i$
Third pole	-72.80
Fourth pole	-10.46
Fifth pole	-0.46
Sixth pole	-5.54×10^{-5}

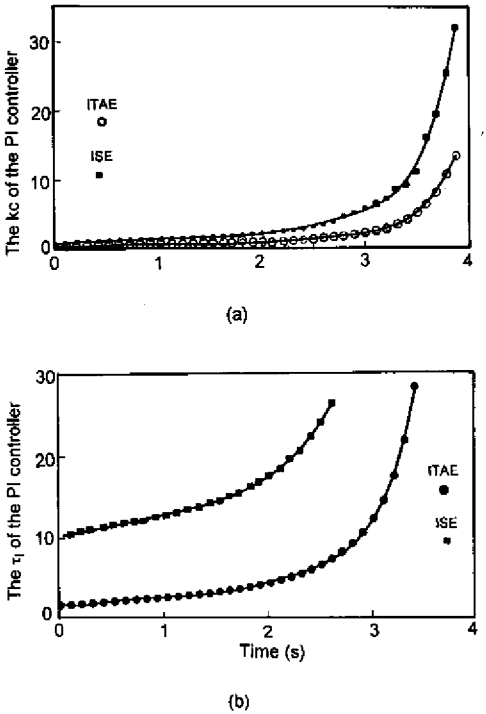

Figure 3. (a) PI Controller gain, (b) PI Controller gain integral time constant.

invariant system and can be used for control purposes. Comparison of non-linear and linear models shows that a piece-wise linearization should be used due to time-varying and non-linearity of the system. This suggests that at least every 0.1 second the model parameters should be up-dated due to time-varying and non-linearity of the system. This is different and a new approach in comparison to usual adaptive control strategy such as one used by Vega et al. [16]. Gao et al. [10] used on-line recursive identification method and applied successfully to control cavity pressure.

However, this approach does not give any physical sight to calculated parameter. Furthermore, in usual adaptive method, a predefined transfer function is assumed. In this paper, however, a physically-based transfer function is derived and adaptation is according to changing nature of process model. This transfer function is a sixth order and has one zero and six poles. Table 1 gives the numerical values of a typical transfer function for i_s 40% and i_R 15% at time 2.3 s.

It is obvious that the only zero of the system is really close to the sixth root. They essentially cancel each other and the transfer function will be a fifth order system without any zero:

$$P_c(s) = \frac{n_1}{\tau(s-r_1)(s-r_2)(s-r_3)(s-r_4)(s-r_5)} T(s) \qquad (5)$$

The fifth order transfer function is used to conduct control studies.

Transition from Filling-to-packing

The filling-to-packing transition should take place at an appropriate time which reflects the completion of the filling phase. Identification of the filling-to-pack-

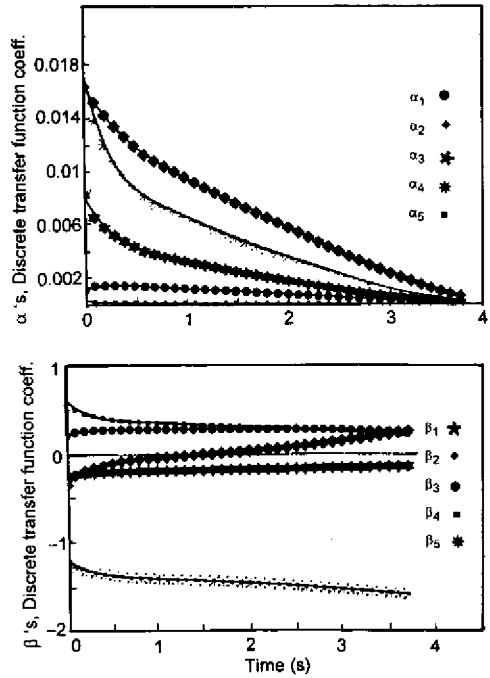

Figure 4. Discrete transfer function coefficients.

ing transition has an important influence on the proper control of cavity pressure during packing [17]. Switching with regard to time derivative of cavity pressure is chosen for this study.

CONTROLLER SELECTION

Packing Pressure Profile

Some researchers have used step change in cavity pressure as the transition from the final value in the filling phase to the final value of the packing phase [12, 18]. However, this pressure profile is unrealistic and causes an inappropriate overshoot. Gao [14] suggested an exponential form. This profile showed an acceptable transition from the end of the filling phase to final value of the cavity pressure during packing. Hence, the pressure profile during the packing phase is approximated by:

$$P_c = P_{FE} + (P_{PE} - P_{FE}) \times (1 - e^{-\frac{t-t_i}{\tau_i}}) \quad (6)$$

where P_{FE} and P_{PE} are the pressure values at the end of filling and packing, respectively; t_i is the filling time; τ_i is the time constant of the packing pressure profile.

Controller Design

The PI and PID controllers are designed. However, the transfer function parameters are changed continuously. Hence, the controllers are adaptive in this implementation because the transfer function parameters change, so do the controllers parameters. There are various methods for the best tuning. One of the widely used controller settings are found by applying time-integral performance criteria. The integral of the time-weighted absolute error (ITAE) and integral of the square error (ISE) methods are used in this work. The form of these integrals are:

$$ITAE = \int_0^\infty t \, |e(t)| \, dt \quad (7a)$$

$$ISE = \int_0^\infty e^2(t)dt \quad (7b)$$

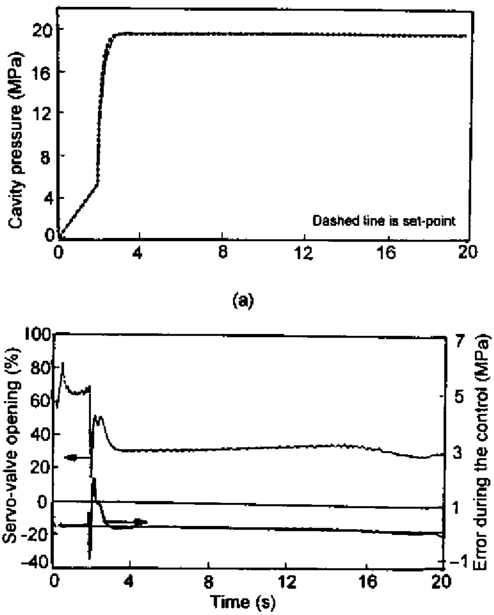

Figure 5. PI Controller tuned with ITAE criteria. (5a) cavity pressure; (5b) servo-valve command and cavity pressure error.

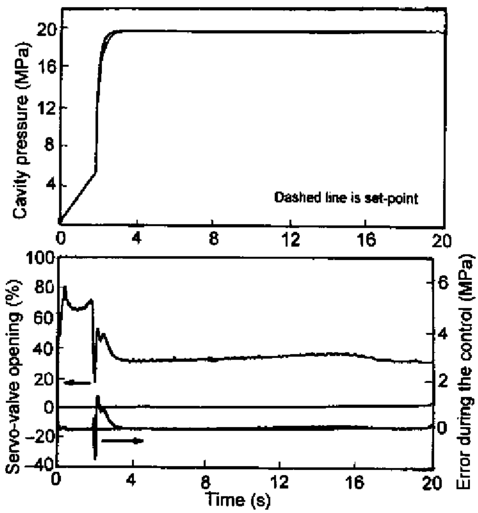

Figure 6. PI Controller tuned with ISE criteria.

The best controller parameters are found by minimizing the value of integral with respect to k_c and τ_i. The fminu function, unconstrained multi-variable function in optimization toolbox of MATLAB, was used to calculate the optimal settings. Figure 3 shows the PI controller parameters versus time, for two criteria. Zero corresponds to the time that the polymer melt fills the mould cavity. The controller gain increases for both tuning methods which is due to decrease in process gain. The integral time constant increases too. Despite the different results the performances of two controllers are very good.

The next controller which studied is internal model control (IMC) [19]. The IMC design method is based on an assumed process model and relates the controller setting to the model parameters in a straightforward manner. The IMC approach has two important advantages: (1) the model uncertainty is taken into account; (2) it takes care of trading off between control system performance and control system robustness.

To design the IMC controller, the discretized process model is used. The discretized process model with zero-order hold is:

$$G(z) = \frac{\alpha_1 z^{-1} + \alpha_2 z^{-2} + \alpha_3 z^{-3} + \alpha_4 z^{-4} + \alpha_5 z^{-5}}{1 + \beta_1 z^{-1} + \beta_2 z^{-2} + \beta_3 z^{-3} + \beta_4 z^{-4} + \beta_5 z^{-5}} \qquad (8)$$

The relationships for α_1, α_2, α_3, α_4, α_5, β_1, β_2, β_3, β_4, and β_5, are given in appendix B. The z is the discretization variable or backward variable and therefore, eqn (8) is called z-transform transfer function. The numerical values of α's and β's are graphically shown in Figure 4 which are calculated off-line.

The points, in Figure 4, are calculated for every 0.1 s using process model, and solid lines are least-squares fit to the points for use in the control algorithm. According to model, after 4 s there is not any change in parameters, although, duration of the packing phase is around 20 s. Then, the final IMC controller is:

$$G_c(z) = \frac{1-\alpha_{IMC}}{1-z^{-1}} \times \frac{1 + \beta_1 z^{-1} + \beta_2 z^{-2} + \beta_3 z^{-3} + \beta_4 z^{-4} + \beta_5 z^{-5}}{\alpha_1 + \alpha_2 z^{-1} + \alpha_3 z^{-2} + \alpha_4 z^{-3} + \alpha_5 z^{-4}} \qquad (9)$$

where a_{IMC} is the low pass filter parameter and to be determined experimentally in this research work.

RESULTS AND DISCUSSION

A series of experiments are devised to test the ability and performance of the controllers. The controllers are applied digitally using the micro-computer system. All the softwares are written in C-language running under QNX operating system. The results for the PI controller tuned with the ITAE are shown in Figure 5. Figure 5a shows the measured cavity pressure and the desired set-point. The ramp portion for times less than 2 s belongs to the filling phase that is controlled by the IMC controller. The cavity pressure begins to drop at about 20–21 s due to gate freezing. Thus, only the first 20 s of the cycle is shown in the figure. The experimental cavity pressure tracks the set-point very well. In the beginning of the packing phase, the cavity pressure is below the set-point and the controller is not fast. Figure 5b shows the command sent to the servo-valve and the measured error. The error versus time is a good indicator of controller performance. It takes approximately 2 s for the error to decrease to approximately zero. The command to the servo-valve is not constant due to the controller's action. The maximum error is 1.0 MPa and it occurs in the beginning of the packing phase where there is a step-like jump in cavity pressure. Figure 6 shows the performance of the PI controller tuned with ISE criterion. The measured cavity pressure is below the set-point. The performance of this controller seems to be better than the PI controller tuned with ITAE, and long time tracking is superior to the one tuned with ITAE. Figures 7 and 8 show the PID controller performances tuned with ITAE and ISE criteria, respectively. Both controllers show very good performances. There is very little overshoot observed in the PID controller's results, which is not seen in the case of the PI controller. This can be attributed to the presence of derivative action in the PID controller.

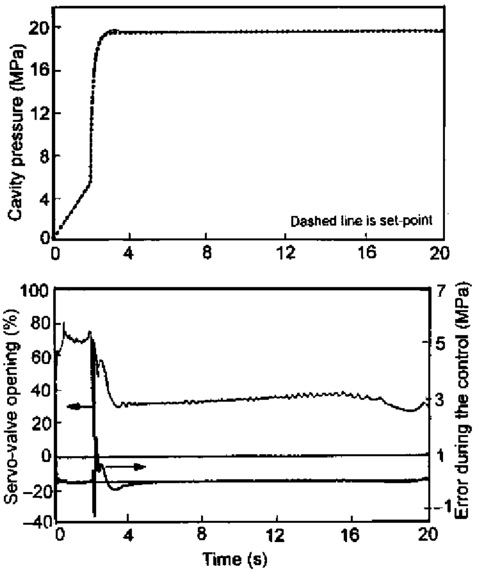

Figure 7. PID Controller tuned with ITAE criteria.

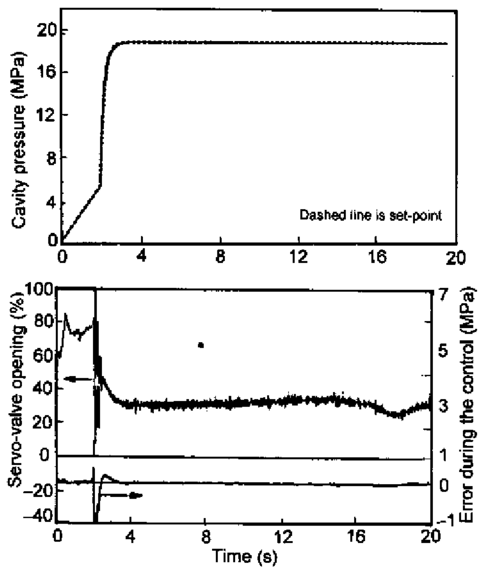

Figure 9. IMC Controller.

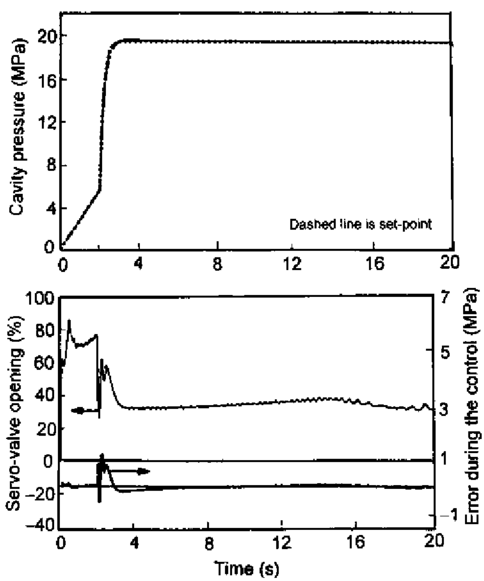

Figure 8. PID Controller tuned with ISE criteria.

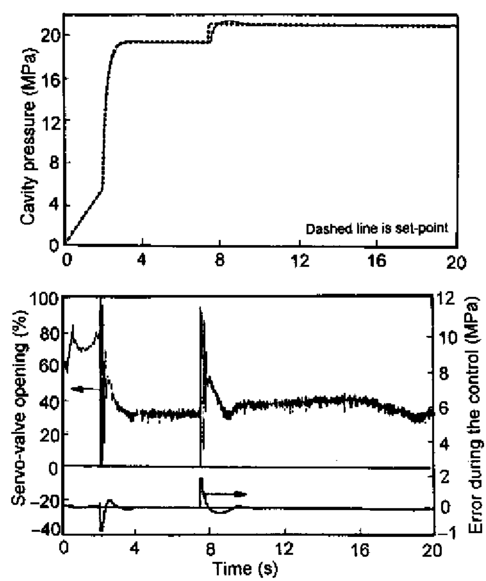

Figure 10. IMC Controller performance to a positive step.

Table 2. The comparison in the performances of different controllers.

Controller type	Average error	Maximum error	Standard deviation
IMC	–0.0089	1.0877	0.1367
PI, ITAE	0.0716	1.3728	0.2015
PI, ISE	0.0269	1.7692	0.1699
PID, ITAE	0.0049	1.1269	0.1618
PID, ISE	0.0020	1.6496	0.1488

Long time tracking ability of both controllers is effective. Figure 9 shows the performance of the IMC controller. The value of a_{IMC} is experimentally determined to be 0.65. The performance of the IMC controller is very good. There is no overshoot or under set-point effect which are seen with the PI and PID controllers. The long time tracking ability of this controller is also very good. However, it appears that in the beginning of the packing phase the controller reacts too quickly. Thus, there is evidence of saturation in controller performance. Over the long term, the command to the servo-valve fluctuates because the IMC controller reacts even to small errors.

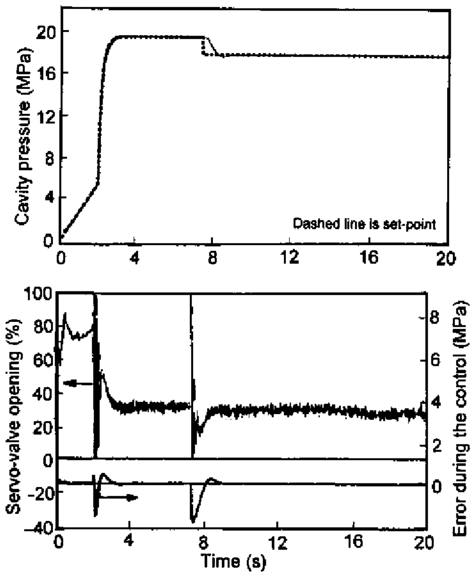

Figure 11. IMC Controller performance to a negative step.

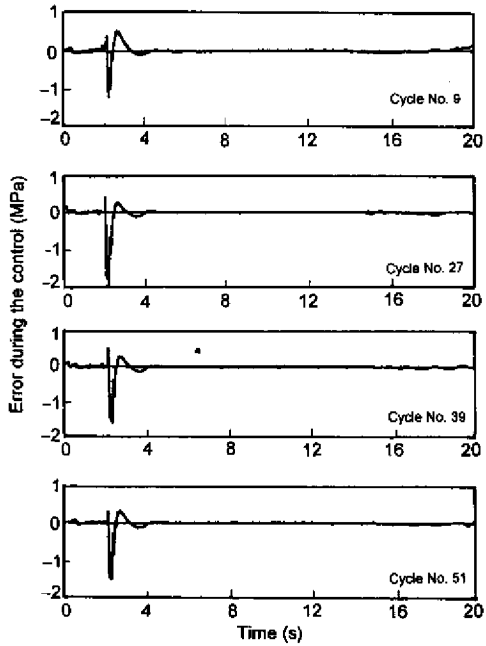

Figure 12. Errors of IMC controller in different cycles.

However, the magnitude of this fluctuation is only 10% opening. The open loop cavity pressure [13] shows a substantial change around 15 s which is corrected by closed loop performance.

For a better quantitative comparison between controller performances, the average error, maximum error, and standard deviation of error for each controller are given in Table 2. The average errors are close to zero, and all of them are positive except for the IMC controller. This is because of the fast reaction of the IMC controller in the beginning of the packing phase. The PI and PID controllers tuned with ISE criteria have the largest error. However, the PID controller tuned with ISE has the smallest average error. The IMC controller has the smallest standard deviation. It should be mentioned that all the controllers have small standard deviation and they have effective control over the cavity pressure. Therefore,

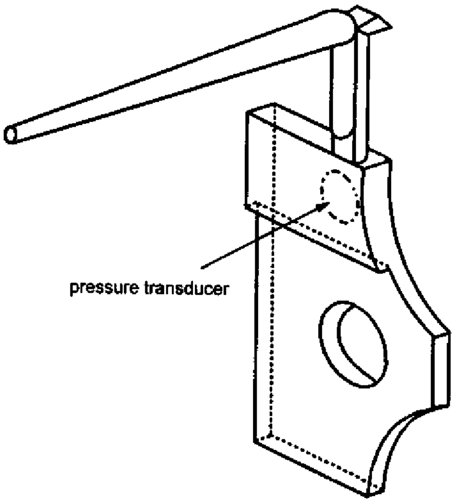

Figure 13. Complex mould cavity.

all of them are satisfactory. The IMC controller is slightly superior, compared to others, for the above reasons.

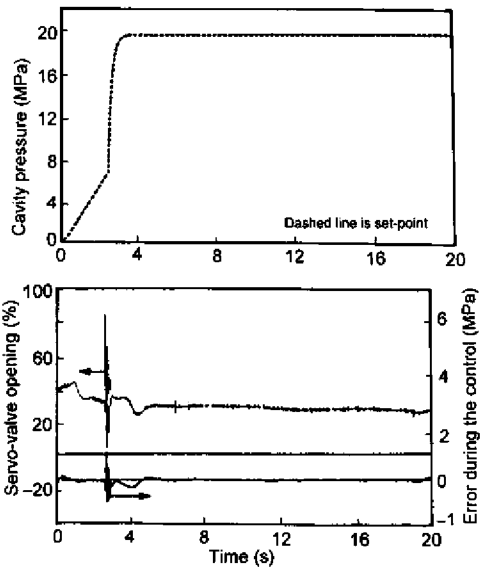

Figure 14. IMC Controller with complex mould cavity.

Some step set-points changes are applied to the system to study the effectiveness of the response of the IMC controller. Figure 10 shows a step change in set-point. The step size is 1.68 MPa. The cavity pressure response shows a very small overshoot followed by short transition time which ends in less than one second. Figure 11 shows the controller performance applied to negative step change in set-point. The transition duration is less than 1 second with some small overshoot.

A mass production of moulded articles is done to study effectiveness of controller in long term. For the sake of brevity, error of some different cycles are shown separately in Figure 12. The controller effectiveness does not deteriorate during this period.

The last part of experiment, for polyethylene, was to apply the IMC controller on a cavity with irregular shape, shown in Figure 13. Figure 14 shows the result of the experiment. The cavity pressure follows the set-point in spite of small overshoot. The value of a_{IMC} in this case was 0.85.

CONCLUSION

The dynamics and control of cavity pressure of the packing phase is studied. A physically based model is used to study the dynamics of the process. Consequently, a linear transfer function with time varying parameters is derived to continue the control studies. Different controllers with distinct tuning criteria are designed. The results are adaptive PI and PID controllers.

Experimental results indicate that these controllers yield accurate results for the injection moulding of high density polyethylene. Change of polymer affects the cavity pressure profile, although controller can handle it. The IMC controller shows excellent controller action for the process. Applying the controller to an irregular cavity shows its ability for controlling of complex and commercial product properties. In the best knowledge of authors, this is the first successful control of injection moulding process using physically-based model.

NOMENCLATURE

a Defined in eqn (11)
$a_{1,2}$ Defined in eqn (21)
a_{IMC} IMC controller parameter
A Defined in eqn (16)
A_h Injection cylinder cross area
A_n Nozzle cross area
b Defined in eqn (11)
$b_{1,2}$ Defined in eqn (21)
B Defined in eqn (17)
c_1 Defined in eqn (22)
d Defined in eqn (11)
d_1 Defined in eqn (22)
D Defined in eqn (18)
e Defined in eqn (11)
e_1 Defined in eqn (22)
$e(t)$ Error at time t
f Defined in eqn (11)
f_1 Defined in eqn (22)
f_i Low pass filter transfer function
F_{1-7} Defined in eqn (2)
g Defined in eqn (10)
G_c Controller transfer function
G_c^* Internal model controller
$\check{G}$ Process transfer function
$\check{G}_+$ Unstable part of process transfer function
$\check{G}_-$ Stable part of process transfer function
H Hydraulic pressure deviation
i Defined in eqn (12)
i_R Relief servo-valve opening (%)
i_S Supply servo-valve opening (%)
j Defined in eqn (12)
k Defined in eqn (12)
k_c Controller gain
k_{ct} Pressure drop coefficient between the nozzle and cavity gate
k_r Radius ratio of screw to nozzle
k_t Total pressure-flow coefficient in polymer delivery system
k_v Servo-valve constant
l Defined in eqn (13)
l_0 Initial length of screw
m Defined in eqn (13)
M Ram screw system mass
n Power law index defined in eqn (13)
$n_{1,2}$ Defined in eqn (4)
N The nozzle pressure deviation
p Defined in eqn (13)
P_c Cavity pressure deviation
P_{gate} Cavity gate pressure
P_h Hydraulic pressure
P_n Nozzle pressure
P_s Supply pressure
q_r Flow rate through relief servo-valve
q_s Flow rate through supply servo-valve
Q_r Deviation of flow through relief valve
Q_s Deviation of flow through supply valve
r_{1-4} Roots of denominator in eqn (4)
R Deviation of relief valve opening (%)
R_n Nozzle radius
s Inverse of power law index 1/n and Laplace transform variable
t Time
T Deviation of supply valve opening (%)
v_z Ram velocity
V Ram velocity deviation
V_h Injection cylinder volume
V_{h0} Initial volume of injection cylinder
V_n Nozzle volume
V_{n0} Initial volume of the nozzle
z Ram displacement
Z Ram displacement deviation
α_{1-5} Discrete transfer function coefficients
β_{1-5} Discrete transfer function coefficients
β_h Hydraulic oil bulk modulus
β_p Polymer bulk modulus
η_0 Polymer viscosity at $\dot{\gamma}=1$
τ Servo-valve time constant
τ_D Derivative time constant
τ_I Integral time constant

Appendix A: Development of Linearized Model

Prior to linearization deviation variables are defined as:

$$T = i_s - i_{s0},\ R = i_R - i_{R0},\ P_c = P_{gate} - P_{gate0},\ Q_r = q_r - q_{r0},\ Q_s = q_s - q_{s0},\ H = P_h - P_{h0},\ Z = z - z_0,\ V = v_z - v_{z0},$$

$$N = P_n - P_{n0} \tag{10}$$

The parameters a through f are derived as:

$$a = \frac{k_v i_{R0}}{2\sqrt{P_{h0}}},\quad b = k_v\sqrt{P_{h0}},\quad c = \frac{k_v i_{S0}}{2\sqrt{P_s - P_{h0}}},\quad d = k_v\sqrt{P_s - P_{h0}},\quad e = \frac{\beta_h}{V_{h0} + A_h z_0},\quad f = \frac{\beta_h A_h (q_{s0} - q_{r0} - A_h v_z)}{(V_{h0} + A_h z_0)^2} \tag{11}$$

The parameters g through k are derived as:

$$g = \frac{A_h}{M}\qquad i = \frac{2\pi\eta_0 R_n^{1-n}(s-1)^n v_{z0}^n}{M(k_r^{1-s} - 1)^n}\qquad j = \frac{2\pi\eta_0 R_n^{1-n}(s-1)^n (L_0 + z_0) n v_{z0}^{(n-1)}}{M(k_r^{1-s} - a)^n}\qquad k = \frac{A_n}{M} \tag{12}$$

The parameters l through p are derived as:

$$l = \frac{\beta_p A_n \left(A_n v_{z0} - \sqrt[n]{\dfrac{P_{n0} - P_{gate0}}{k_t}}\right)}{(v_{z0} - A_n z_0)^2}\qquad m = \frac{\beta_p A_n}{V_{z0} - A_n z_0}$$

$$n = \beta_p \frac{\sqrt[n]{\dfrac{P_{n0} - P_{gate0}}{k_t}}}{(V_{z0} - A_n z_0) n (P_{n0} - P_{gate0})}\qquad p = \beta_p \frac{\sqrt[n]{\dfrac{P_{n0} - P_{gate0}}{k_t}}}{V_t n (P_{n0} - P_{gate0})} \tag{13}$$

Appendix B: Discretization of G(s)

The fifth order transfer function is expanded via partial fractions, as:

$$G(s) = \frac{n_1}{\tau(s - r_1)(s - r_2)(s - r_3)(s - r_4)(s - r_5)} = \frac{n_1}{\tau}\left[\frac{As + B}{s^2 - 2as + a^2 + b^2} + \frac{C}{s - r_3} + \frac{D}{s - r_4} + \frac{E}{s - r_5}\right] \tag{14}$$

where:

$$r_1 = a + bi,\quad r_2 = a - bi \tag{15}$$

and A is:

$$A = \frac{(r_4 - r_5)(r_5^2 - 2ar_5 + a^2 + b^2)(r_4^2 - 2ar_4 + a^2 + b^2)}{\text{DEN.}} + \frac{(r_5 - r_3)(r_5^2 - 2ar_5 + a^2 + b^2)(r_3^2 - 2ar_3 + a^2 + b^2)}{\text{DEN.}}$$

$$+ \frac{(r_3 - r_4)(r_3^2 - 2ar_3 + a^2 + b^2)(r_4^2 - 2ar_4 + a^2 + b^2)}{\text{DEN.}} \tag{16}$$

and B is:

$$B = \frac{(2a - r_3)(r_5 - r_4)(r_5^2 - 2ar_5 + a^2 + b^2)(r_4^2 - 2ar_4 + a^2)}{DEN.}$$

$$+ \frac{(2a - r_4)(r_3 - r_5)(r_5^2 - 2ar_5 + a^2 + b^2)(r_3^2 - 2ar_3 + a^2)}{DEN.} + \frac{(2a - r_5)(r_4 - r_3)(r_3^2 - 2ar_3 + a^2 + b^2)(r_4^2 - 2ar_4 + a^2)}{DEN.} \qquad (17)$$

and C, D, and E are:

$$C = \frac{(r_5 - r_4)(r_5^2 - 2ar_5 + a^2 + b^2)(r_4^2 - 2ar_4 + a^2 + b^2)}{DEN.}$$

$$D = \frac{(r_3 - r_5)(r_5^2 - 2ar_5 + a^2 + b^2)(r_3^2 - 2ar_3 + a^2 + b^2)}{DEN.}$$

$$E = \frac{(r_4 - r_3)(r_3^2 - 2ar_3 + a^2 + b^2)(r_4^2 - 2ar_4 + a^2 + b^2)}{DEN.} \qquad (18)$$

and the denominator, DEN., is:

DEN. =

$$(r_5 - r_4)(r_5^2 - 2ar_5 + a^2 + b^2)(r_4^2 - 2ar_4 + a^2 + b^2)[-r_3r_4r_5(2a - r_3) + r_4r_5(a^2 + b^2)]$$

$$+ (r_3 - r_5)(r_5^2 - 2ar_5 + a^2 + b^2)(r_3^2 - 2ar_3 + a^2 + b^2)[-r_3r_4r_5(2a - r_4) + r_3r_5(a^2 + b^2)]$$

$$+ (r_4 - r_3)(r_3^2 - 2ar_3 + a^2 + b^2)(r_4^2 - 2ar_4 + a^2 + b^2)[-r_3r_4r_5(2a - r_5) + r_3r_4(a^2 + b^2)] \qquad (19)$$

The discretized form of the transfer function with zero order hold is:

$$G(z) = \frac{n_1}{\tau}\left[\frac{b_1z^{-1} + b_2z^{-2}}{1 + a_1z^{-1} + a_2z^{-2}} + \frac{d_1z^{-1}}{1 + c_1z^{-1}} + \frac{f_1z^{-1}}{1 + e_1z^{-1}} + \frac{h_1z^{-1}}{1 + g_1z^{-1}}\right] \qquad (20)$$

where:

$$a_1 = -2e^{a\Delta t}\cos b\Delta t \qquad a_2 = e^{2a\Delta t} \qquad b_1 = \frac{-B}{a^2 + b^2} - e^{a\Delta t}\left[\frac{A}{b}\sin b\Delta t + \frac{B}{b}\frac{a}{a^2 + b^2}\sin b\Delta t - \frac{B}{a^2 + b^2}\cos b\Delta t\right]$$

$$b_2 = \frac{Ae^{a\Delta t}}{a^2 + b^2}\left[\frac{B}{A}e^{a\Delta t} - \frac{B}{A}\cos b\Delta t - \frac{a^2 + b^2 + \frac{aB}{A}}{b}\sin b\Delta t\right] \qquad (21)$$

and

$$c_1 = -e^{r_3\Delta t} \qquad d_1 = \frac{C}{-r_3}(1-e^{r_3\Delta t}) \qquad e_1 = -e^{r_4\Delta t} \qquad f_1 = \frac{D}{-r_4}(1-e^{r_4\Delta t}) \qquad g_1 = -e^{r_5\Delta t} \qquad h_1 = \frac{E}{-r_5}(1-e^{r_5\Delta t}) \tag{22}$$

Finally, rearrangement of this equation, with a zero-order hold, gives the discretized transfer function:

$$G(z) = \frac{\alpha_1 z^{-1} + \alpha_2 z^{-2} + \alpha_3 z^{-3} + \alpha_4 z^{-4} + \alpha_5 z^{-5}}{1 + \beta_1 z^{-1} + \beta_2 z^{-2} + \beta_3 z^{-3} + \beta_4 z^{-4} + \beta_5 z^{-5}} \tag{23}$$

The relationships for α_1, α_2, α_3, α_4, α_5 are:

$$\alpha_1 = \frac{n_1}{\tau}(b_1 + d_1 + f_1 + h_1)$$

$$\alpha_2 = \frac{n_1}{\tau}(b_1g_1 + b_1e_1 + b_1c_1 + b_2 + d_1g_1 + d_1e_1 + d_1a_1 + f_1g_1 + f_1c_1 + f_1a_1 + h_1e_1 + h_1c_1 + h_1a_1) \tag{24}$$

and

$$\alpha_3 = \frac{n_1}{\tau}(b_1e_1g_1 + b_1c_1g_1 + b_1c_1e_1 + b_2g_1 + b_2e_1 + b_2c_1 + d_1e_1g_1 + d_1a_1g_1 + d_1a_1e_1 + d_1a_2 + f_1c_1g_1 + f_1a_1g_1$$

$$+ f_1a_1c_1 + f_1a_2 + h_1c_1e_1 + h_1a_1e_1 + h_1a_1c_1 + h_1a_2) \tag{25}$$

and

$$\alpha_4 = \frac{n_1}{\tau}(b_1c_1e_1g_1 + b_1e_1g_1 + b_2c_1g_1 + b_2c_1e_1 + b_2c_1e_1 + d_1a_1e_1g_1 + d_1a_2g_1 + d_1a_2e_1 + f_1a_1c_1g_1 + f_1a_2g_1$$

$$+ f_1a_2c_1 + h_1a_1c_1e_1 + h_1a_2e_1 + h_1a_2c_1)$$

$$\alpha_5 = \frac{n_1}{\tau}(b_2c_1e_1g_1 + d_1a_2e_1g_1 + f_1a_2c_1g_1 + h_1a_2c_1e_1) \tag{26}$$

The relationship for β_1, β_2, β_3, β_4, and β_5 are:

$$\beta_1 = g_1 + e_1 + c_1 + a_1 \qquad \beta_2 = e_1g_1 + c_1g_1 + c_1e_1 + a_1g_1 + a_1e_1 + a_1c_1 + a_2$$

$$\beta_3 = c_1e_1g_1 + a_1e_1g_1 + a_1c_1g_1 + a_1c_1e_1 + a_2g_1 + a_2e_1 + a_2c_1$$

$$\beta_4 = a_1c_1e_1g_1 + a_2e_1g_1 + a_2c_1g_1 + a_2c_1e_1 \qquad \beta_5 = a_2c_1e_1g_1 \tag{27}$$

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