A Transient Finite Element Model of Mixing of Rubber Compounds in a Banbury Mixer

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ABSTRACT

The aim of this paper is to develop a mathematical model for the simulation of the flow of elastomeric materials inside a typical Banbury mixer. The model combines the non-Newtonian behaviour of rubber compound with non-isothermal flow regime in a transient condition. The set of the governing equations are solved using the finite element method. The momentum equations are solved using continuous penalty method and the energy equation is solved employing the Galerkin method. A streamline upwinding technique is used in the solution scheme of the energy equation to provide convergent and stable results for high Peclet number flow. The solution of the transient equations is performed using implicit \(t\) time stepping method. The overall solution strategy is based on the de-coupling of the flow and energy equation and use of the Picard iterative scheme. The results of the simulation include the velocity, pressure and temperature distributions and the variation of these parameters with time. Comparison of these results with the available experimental data confirms the general validity of the development model.

Key Words: mathematical modeling, finite element method, simulation, internal mixers, elastomers

INTRODUCTION

The process of mixing of elastomeric materials with the compounding ingredients such as carbon black, oil and other additives in internal mixers is considered to be the most important step in determining the properties of the final product. Internal mixers have been used as a dominant rubber mixing device for more than a century. It was, however, only in the twentieth century that the widespread use of rubber compounds resulted in extensive use of internal mixers. It should be pointed out that there are two types of internal mixers used by rubber industries namely the Banbury mixer and the Francis Shaw Intermix[1]. These types of mixers differ from each other due to the shape of rotors and speeds at which these rotors rotate.

In this research work we simulate the flow inside the Banbury mixer. The development of this mixer to its current form is attributed to the Birmingham Iron Foundry (later Farrel Corporation). A Banbury mixer consists of a mixing chamber shaped like the symbol used for digit eight with a rotor in each part. Each rotor has two or four wings on its surface. The rotors are operated at different speeds. The compounding ingredients
are fed to the mixing chamber through a vertical chute in which an air or a hydraulic driven ram is located. The homogenized mixture is discharged through a drop-door at the bottom.

There have been numerous attempts in the past to develop mathematical models in order to study the flow and mixing in internal mixers. This is because a mathematical model can predict stress, velocity and temperature distributions in its domain. This in turn can be used to optimize the design or the operating conditions and thus reducing the cost and the energy consumption.

Background

The early flow analysis in an internal mixer can be attributed to Bergen, et al.[2], which dates back to late 1950s. It is a very simple analytical model for the shearing flow of a viscous fluid between the blade and the chamber wall. Tadmor[3] also presented a more sophisticated but analytical model based on the assumption of isothermal, Newtonian fluid for the flow between two long concentric cylinders with a short low clearance and negligible curvature. Following these early models, many investigators tried to use simple numerical methods to develop more complicated models.

White et al.[4] used the lubrication theory and modelled the flow in an internal mixer based on the assumption of isothermal flow regime and Newtonian fluid. The finite difference method was used to solve the derived set of the equations. Kim et al.[5] and White et al.[6] improved this model to include the effect of non-Newtonian behaviour. Kim et al.[7] developed a 3-dimensional non-Newtonian non-isothermal model based on the lubrication theory to predict the longitudinal pumping effect.

Hu et al.[8-9] introduced a more rigorous approach than the previous model based on the lubrication theory extended to cylindrical coordinates which permits full consideration of curvature of the flow domain. This analysis, however, was limited to Newtonian fluids and fully filled chamber.

It is well known that during the mixing of elastomeric materials in an internal mixer, a very complicated 3-dimensional flow field with multiple free surface is established. The rheological behaviour of rubber compounds also changes in response to mixing process and cannot be adequately described even using apparently complicated equations. Additional difficulties arise from the incorporation of boundary conditions affecting mixer flow field. Consequently there has been various attempts by different researchers who have tried to develop mathematical models of mixing process using advanced numerical technique such as the finite element method.

Cheng et al.[10-14] used FIDAP (a fluid dynamic package based on the finite element method) to analyze the flow and to study the dispersive mixing in a Banbury mixer (type B). The analyses were performed based on 2-dimensional, steady state and transient along with the isothermal and non-isothermal flow regimes, as well as the non-Newtonian (power-law) fluid conditions.

Yang et al.[15] developed a 3-dimensional model for the flow of rubber inside a BB-2 type Banbury mixer in order to study the material motion in the direction normal to the rotor wings using FIDAP package. Wong et al.[16], in a recent paper, started the study of the distributive mixing by using their previously mentioned model based on FIDAP.

Nassehi, et al.[17] performed a wide series of finite element analysis for rubber mixing and carried out finite element analysis of steady state, non-isothermal power-law fluids within a 2-dimensional domain representing real mixer geometry. A fully filled chamber was selected. In order to study the effect of viscoelastic behaviour of rubbery materials, Nassehi et al.[18] used a new finite element algorithm to simulate a 2-dimensional steady viscoelastic flow inside an internal mixer. The main aim of this study was to find the limiting stress at which the material is prevented from entering the narrow gap between the blade and chamber wall. Nassehi et al.[19] attempted to apply a viscometric constitutive equation known as (CEF) model (a simple viscoelastic model) to the simulation of predominantly circumferential flow in a 2-dimensional representation of an internal mixer using polar coordinate system.
Present Study
The present research work is devoted to the mathematical modelling of flow of an elastomeric material inside the Banbury mixer. The set of the governing equations are solved using finite element technique via an in-house developed computer code. This computer program is used to simulate the flow inside a typical mixer.

MATHEMATICAL MODEL
The governing equations of the transient, non-isothermal flow of an incompressible non-Newtonian fluid in a two dimensional Cartesian coordinate system in the absence of body force are given as the following equation:

-continuity equation:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

(1)

-x component of the momentum equation:

\[
\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}
\]

(2)

-y component of the momentum equation:

\[
\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x}
\]

(3)

-energy equation:

\[
\rho C_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + Q
\]

(4)

In these equations \(u\) and \(v\) are the \(x\) and \(y\) components of the velocity vector, \(p\) is the pressure, \(T\) is the temperature, \(\rho\) is the material density, \(C_p\) is the heat capacity of the rubber, \(k\) is the thermal conductivity of the rubber, \(\tau_{xx}, \tau_{xy}, \tau_{yy}\) are the components of viscous stress tensor \(\tau\) which are given for a generalized Newtonian model in terms of rate-of-deformation tensor \(\gamma\) by:

\[
\tau = \eta\gamma
\]

(5)

where \(\eta\) is shear dependent non-Newtonian viscosity of the fluid. The rate-of-deformation tensor is defined as:

\[
\gamma = \begin{bmatrix}
2\frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \\
\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} & 2\frac{\partial v}{\partial y}
\end{bmatrix}
\]

(6)

Viscosity \(\eta\) in the present study is given by a power-law type equation known as Carreau model [20]:

\[
\eta = \eta_0 \left(1 + \lambda_0 \left(\frac{1}{2} - I_2\right)\right)^{(n-1)/2} e^{\kappa(T-T_0)}
\]

(7)

where \(\eta_0\) is the consistency of the fluid, \(n\) is the power-law index, \(T_0\) is a reference temperature, \(b\) is the temperature sensitivity coefficient, \(\lambda_0\) is the relaxation time reflecting the elastic behaviour of the fluid and \(I_2\) is the second invariant of rate-of-deformation tensor defined as:

\[
I_2 = (\frac{\partial u}{\partial x})^2 + 2(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})^2 + (\frac{2\partial v}{\partial y})^2
\]

(8)

The source term \(Q\) in the energy equation(4) represents the viscous heat dissipation effect and is expressed by:

\[
Q = \frac{1}{2} \eta I_2
\]

(9)

FINITE ELEMENT FORMULATION
Flow Equations
The finite element formulation of flow equations can be based on either a pressure-velocity (mixed or \(u-v-p\)) scheme or the use of the penalty methods. In the present study we have selected the penalty technique because it produces a more compact set of working equations, thus, reducing the required computer storage and computational cost. Furthermore, there is some evidence which shows that for highly viscous fluid the penalty method gives more accurate solution than the mixed method. The pressure field is found by a secondary calculation based on the variational recovery method[21].
The basic step in the penalty formulation is the elimination of the pressure term in momentum equations using:
\[
p = -\lambda \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)
\]  
(10)

where \( \lambda \) is a penalty parameter. It can be shown that if we choose \( \lambda \) to be a relatively large number, the continuity equation will be satisfied. It is recommended that the value of \( \lambda \) be considered as a function of viscosity to ensure uniform continuity enforcement in non-Newtonian problems [22]. Therefore equation (10) is written as:
\[
p = -\lambda^* \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)
\]  
(11)

where:
\[
\lambda^* = \eta \lambda
\]  
(12)

In this equation \( \eta \) is the local viscosity and \( \lambda \) is a large positive number (say, \( 10^{19} \)). There are two ways of using the relation (11) in order to eliminate the pressure in the momentum equations. These are known as the continuous and the discrete penalty methods [22]. In the continuous penalty method the relation (11) is directly incorporated into the momentum equations while in the discrete penalty method the modified continuity equation (11) is treated in the usual Galerkin finite element method to give the discretized form. In the present research work, the derived set of working equations are based on the use of continuous penalty method.

Following the basic procedures of the weighted residuals finite element techniques, the momentum equations are multiplied by weight functions and the integral of the obtained residuals over the solution domain (\( \Omega_e \)) are set to be zero, thus we obtain:
\[
f_\omega \left[ \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + \frac{\partial p}{\partial x} - \frac{\partial}{\partial x} \left( 2\eta \frac{\partial u}{\partial x} \right) - \frac{\partial}{\partial y} \left( \eta \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \right) \right) \right] d\Omega = 0
\]  
(13)
\[
f_\omega \left[ \rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) + \frac{\partial p}{\partial y} - \frac{\partial}{\partial x} \left( 2\eta \frac{\partial v}{\partial x} \right) - \frac{\partial}{\partial y} \left( \eta \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \right) \right) \right] d\Omega = 0
\]  
(14)

Substituting the equation (11) into above equations and applying the divergence theorem, we will have the weak variational forms of the governing equations as:
\[
\int_\Omega \left[ \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) \right] w_1 + \left[ 2\eta \frac{\partial u}{\partial x} + \lambda^* \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] w_1 + \left[ \eta \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \right) \right] w_1 \right] \eta w_1 \left( \left[ 2\eta \frac{\partial u}{\partial x} + \lambda^* \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] \right) d\Omega = \int_\Omega \lambda^* \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) d\Omega = \int_\Omega \left( \eta \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \right) \right) d\Omega = 0
\]  
(15)
\[
\int_\Omega \left[ 2\eta \frac{\partial v}{\partial y} + \left( \eta \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \right) \right) \right] w_2 + \left[ 2\eta \frac{\partial v}{\partial y} + \lambda^* \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] w_2 + \left[ \eta \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \right) \right] w_2 \right] \eta w_2 \left( \left[ 2\eta \frac{\partial v}{\partial y} + \lambda^* \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] \right) d\Omega = \int_\Omega \lambda^* \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) d\Omega = \int_\Omega \left( \eta \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \right) \right) d\Omega = 0
\]  
(16)

In the finite element context the solution domain is discretized into a mesh consisting of finite elements (\( \Omega_e \)) and over each element we approximate \( u \) and \( v \) by interpolation functions as:
\[
u = \sum_{i=1}^{s} u_i(t) \Psi_i(x,y), ~ v = \sum_{i=1}^{s} v_i(t) \Psi_i(x,y)
\]  
(17)

where \( \Psi_i(x,y) \) are the interpolation functions and \( u_i(t) , v_i(t) \) are the nodal velocities. Using Galerkin method in which the weight functions are the same as the interpolation functions, the finite element method working equations associated with penalty formulation are derived as:
\[
\begin{bmatrix}
[M^{11}] & 0 \\
0 & [M^{11}]
\end{bmatrix}
\begin{bmatrix}
[u] \\
[v]
\end{bmatrix}
+
\begin{bmatrix}
[K^{11}] & [K^{12}] \\
[K^{21}] & [K^{22}]
\end{bmatrix}
\begin{bmatrix}
[u] \\
[v]
\end{bmatrix}
=
\begin{bmatrix}
[F^1] \\
[F^2]
\end{bmatrix}
\]  
(18)

where:
\[
(M^{11})_{ij} = \int \int \rho \Psi_i \Psi_j \ dx \ dy
\]  
(19)
\[
(k^{11})_{ij} = \int \int (\ \vec{u} \frac{\partial \Psi_j}{\partial x} + \vec{v} \frac{\partial \Psi_j}{\partial y}) \ dx \ dy + \\
\int \int \lambda^* \frac{\partial \Psi_i}{\partial x} \ \frac{\partial \Psi_j}{\partial x} \ dx \ dy + \int \int \lambda^* \frac{\partial \Psi_i}{\partial y} \ \frac{\partial \Psi_j}{\partial y} \ dx \ dy
\]
\[
\frac{\partial \Psi}{\partial x} \, dx \, dy
\]  
(20)

\[
(k_{11})_{ij} = \int f \frac{\partial \Psi_i}{\partial y} \frac{\partial \Psi_j}{\partial x} \, dx \, dy + \int f \frac{\partial \Psi_i}{\partial x} \frac{\partial \Psi_j}{\partial y} \, dx \, dy
\]  
(21)

\[
(k_{21})_{ij} = \int f \frac{\partial \Psi_i}{\partial x} \frac{\partial \Psi_j}{\partial y} \, dx \, dy + \int f \frac{\partial \Psi_i}{\partial y} \frac{\partial \Psi_j}{\partial x} \, dx \, dy
\]  
(22)

\[
\frac{\partial \Psi_i}{\partial y} + \eta \frac{\partial \Psi_i}{\partial x} \frac{\partial \Psi_i}{\partial y} \, dx \, dy + \int f \frac{\partial \Psi_i}{\partial y} \frac{\partial \Psi_j}{\partial x} \, dx \, dy
\]  
(23)

\[
(F_i) = \int \Psi_i(2\eta \frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}) d\Gamma
\]  
(24)

\[
(F_i) = \int \Psi_i(2\eta \frac{\partial v}{\partial y} + \frac{\partial u}{\partial y}) d\Gamma
\]  
(25)

Equation (18) can be written in the compact form as:

\[
[M]{\dot{X}} + [K]{X} = \{F\}
\]  
(26)

where \([M]\) is the mass matrix, \([K]\) is the stiffness matrix, \([F]\) is the right hand side (load vector), \(\dot{X}\) is the first derivative of vector of unknown and \(X\) is the solution vector. In order to obtain a non-trivial solution the penalty terms in the stiffness matrix coefficients must be evaluated using reduced integration. For example, when using 2-point Gauss quadrature must be used to evaluate the terms including penalty parameters while the 3-point Gauss quadrature is used to calculate the non-penalty terms[23].

**Energy Equation**

Applying the Galerkin method to equation (4) and carrying out the integration by parts (divergence theorem), the weak variational form of the energy equations is obtained as:

\[
\int \rho C_p \frac{\partial T}{\partial t} d\Omega + \int \rho C_p (u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}) d\Omega +
\]

\[
\int k \left( \frac{\partial \Psi_i}{\partial x} \frac{\partial \Psi_j}{\partial x} + \frac{\partial \Psi_i}{\partial y} \frac{\partial \Psi_j}{\partial y} \right) d\Omega = \int \Psi_i (k \frac{\partial T}{\partial n}) d\Gamma +
\]

\[
\int \Psi_i Q d\Omega
\]  
(27)

The temperature field over a typical element is approximated in terms of interpolation functions as:

\[
T = \sum_{i=1}^{n} T_i \Psi_i(x,y)
\]  
(28)

Thus the finite element working equations of the energy equation will be given as:

\[
[M] \{\dot{T}\} + ([K] + [K_c]) \{T\} = \{F\}
\]  
(29)

where \([M]\) is the mass matrix, \([K_c]\) and \([K_s]\) are the convection and conduction stiffness matrices, \(\dot{T}\) and \(T\) are the solution vector of the temperature field and its first order derivative, respectively, and \(\{F\}\) is the load vector. These matrices are given by the following relations:

\[
(M)_{ij} = \int f \rho C_p \Psi_i \Psi_j d\Omega
\]  
(30)

\[
(K_c)_{ij} = \int f \rho C_p \left[ u \frac{\partial \Psi_i}{\partial x} + v \frac{\partial \Psi_j}{\partial y} \right] d\Omega
\]  
(31)

\[
(K_s)_{ij} = \int k \left( \frac{\partial \Psi_i}{\partial x} \frac{\partial \Psi_j}{\partial x} + \frac{\partial \Psi_i}{\partial y} \frac{\partial \Psi_j}{\partial y} \right) d\Omega
\]  
(32)

\[
(F)_{i} = \int \Psi_i (k \frac{\partial T}{\partial n}) d\Gamma + \int \Psi_i Q d\Omega
\]  
(33)

The weight functions \(\Psi_i\) have been used for the convection term, \(K_c\) to distinguish it from the \(\Psi_i\) used in other terms. This is due to the very low thermal conductivity of elastomeric materials which results the heat transfer to be essentially by convection mechanism. Mathematically this means that the first order derivatives in equation (4) are dominant. To overcome any numerical instability associated with the treatment of first order derivatives a streamline upwinding approach[24] specially developed for biquadratic element is used. The weight function for convective terms is written as:

\[
w_i = \Psi_i \Phi (u \frac{\partial \Psi_i}{\partial x} + v \frac{\partial \Psi_i}{\partial y})
\]  
(34)
where \( \Phi \) is a multiplier known as upwinding parameter. The value of \( \Phi \) determines the amount of upwinding applied into the solution scheme. The optimum value of \( \Phi \) for two-dimensional biquadratic elements is given as:

\[
\Phi = \frac{\| \lambda_t \| v_t^2 + \| \lambda_n \| v_n^2}{|v|^2} \cdot \frac{1}{|v|}
\]

where \( v \) is the velocity vector, \( v_t \) and \( v_n \) are the velocity components in directions of \( e_t \) and \( e_n \) (unit vectors directed along the mapped local coordinates at integration points) finally \( \lambda_t \) and \( \lambda_n \) are two parameters defined in terms of the mesh Peclet number(25).

Solution of Transient Equations

The previously developed finite element working equations are first order set of differential equations with respect to time. Of those available time integration methods for the solution of these type of equations, we use a well known single time step procedure belonging to \( \theta \) family of approximation methods. In this technique direct numerical integration is applied and a recurrence formula which relates the values of solution vector at time \( t_0 \) to the values of the solution vector at a later time \( (t + \Delta t) \) is obtained. To illustrate this method briefly, let \( (t_0) \) denote a typical time between \( t_n \) and \( t_{n+1} \) so that \( t_0 = t_n + \theta \Delta t \) where \( 0 \leq \theta \leq 1 \).

We write the final form of finite element formulation at time \( t_0 \) as:

\[
[M]{X}_{t_0} + [K]{X}_{t_0} = \{F\}_{t_0}
\]

By introducing the following approximations:

\[
\{\dot{X}\} = \frac{\{X\}_{n+1} - \{X\}_n}{\Delta t}
\]

\[
\{X\} = (1-\theta \{X\}_n + \theta \{X\}_{n+1}
\]

\[
\{F\}_t = (1-\theta \{F\}_n + \theta \{F\}_{n+1}
\]

and substituting the above equations into equation (36), we obtain:

\[
([M] + \theta \Delta t[K])\{X\}_{n+1} = ([M] - (1-\theta)\Delta t[K])\{X\}_n + ((1-\theta)\{F\}_n + \theta \{F\}_{n+1})\Delta t
\]

where \( \{X\}_{n+1} \) on the left hand side is unknown and all of the terms on the right hand side are known. For a given value of \( \theta \), equation (40) represents a recurrence formula in which the nodal values \( \{X\}_{n+1} \) are calculated from the known values of \( \{X\}_n \) at the beginning of the time step. In the present study, we selected the \( \theta = 1 \) (backward or full implicit method), thus, provided an unconditionally stable scheme.

Boundary and Initial Conditions

For momentum equations, it is assumed that the shear stress at walls is less than a critical value, thus, the rubber compound completely sticks to the wall (no-slip). In this case all of the prescribed boundary conditions are either of the first type (essential or Dirichlet boundary conditions) or of the second type (natural or von-Neumann boundary conditions). The flow domain which we study in this research work consists of a fully filled chamber with a Banbury type rotor blade (Figure 1). We assume that the pressure is zero along the inlet and outlet surfaces of the flow domain. Therefore the line integral in the right hand side of the working equations vanishes and the only boundary conditions which need to be imposed are Dirichlet (first type) conditions on the domain walls. The boundary conditions for the energy equation are only of the first type i.e. prescription of temperature values at chamber wall, blade surface and inlet of the flow domain (Figure 1). Zero flux condition is
given at the domain exit. For the initial conditions, it is assumed that all of the materials and boundaries have a zero velocity and uniform temperature equal to 298 K (25°C).

GLOBAL SOLUTION STRATEGY

Using the isoparametric mapping[21] the working equations of the present scheme are cast into a local (natural) coordinate system. The members of the coefficient matrices are then computed for each element by a Gauss quadrature method. The resulting algebraic equations are assembled into a global matrix and after imposing the appropriate set of boundary conditions are solved by a frontal solution algorithm[26]. The presence of the convective terms in the momentum and energy equations as well as the dependency of the local viscosity gradients, make this set of equations nonlinear. Consequently a de-coupled iterative procedure based on the successive substitution method (Picard iteration method)[22] was adopted. In this procedure, at the beginning of the first iteration, the velocity field is set to zero and the temperature field is set to be equal to the initial conditions, and the coefficient matrices are computed and assembled. The global equations are solved to obtain the velocity field. The obtained velocity field in turn is used in the calculation of viscosity and the solution of the energy equation. Using the computed velocity and temperature fields at the end of the first iteration as initial estimates, the procedure is repeated until the velocity and the temperature fields are converged. The convergence criterion used in this work is given by:

$$\sum_{i=1}^{N} \frac{|X_{i}^{r+1} - X_{i}^{r}|^2}{\sum_{i=1}^{N}|X_{i}^{r+1}|^2} \leq \delta$$  \hspace{1cm} (41)$$

where $X_{i}^{r}$ denotes the flow variables (velocity or temperature) at degree of freedom $i$ at iteration cycle of $r$, and $\delta$ is the convergence tolerance. After the solution of the governing equations, the solution procedure is forwarded for a new time step and this is repeated until the end of the number of desired time steps has been reached.

RESULTS AND DISCUSSION

A mixing chamber of 0.1m radius and gap width (distance between the blade tip and the chamber wall) of 0.006m is considered. The chamber wall rotates at 0.2m/sec and the blade is assumed to be stationary. The temperature at both walls (chamber and blade surface) is maintained at 323 K and the temperature at the inlet of the domain is 360 K (Figure 1). The physical and rheological properties of the rubber compound are as:

$$\eta_0 = 2 \times 10^4 \text{ Pa.s}$$
$$n = 0.2$$
$$b = 0.014 K^{-1}$$
$$T_0 = 373 K$$
$$\lambda_0 = 1.15 s$$
$$\rho = 1055 \text{ kg/m}^3$$
$$C_p = 1255 \text{ J/kg.K}$$
$$k = 0.13 \text{ W/m.K}$$

The finite element mesh for this analysis consists of 210 9-noded Lagrangian elements with total number of nodes equal to 923 (Figure 2).

The problem was analyzed under both steady state and transient conditions for a non-isothermal flow regime. For the steady state case, the results
are shown in Figures 3-6. Figure 3(a) shows the velocity vector field in the flow domain. The details of the velocity field at the gap region is drawn in Figure 3(b). The flow streamlines are also presented in Figure 4. As it can be seen, the flow domain is divided into two parts. The first part involves the region between the blade tip and the chamber wall and its associated trailing zone. The flow pattern in this region is similar to drag or Couette flow. The other region, the fluid undergoes a circular motion. These streamlines are in total accordance with the velocity field presented in Figure 3(a,b). The pressure distribution is shown in Figure 5. The highest value for the pressure is about $0.3 \times 10^7$ Pa and it occurs as expected at the vicinity of the blade tip. The experimentally measured pressure at the proximity of the blade is between $0.25 \times 10^7$ and $0.35 \times 10^7$ [27] and as it can be seen it is in good agreement with calculated results. The temperature distribution is presented in Figure 6. The maximum computed temperature is around 371 K which belongs to the region near the inlet where the thickness of rubber compound is high. It is expected that, the low thermal conductivity of the material should result in the rise of temperature at this region.

The problem is also analyzed for transient case. The time increment for this model is selected to be equal to 0.05 s. The variations of pressure and temperature for three nodes located at near to the entrance to the gap, inside and outside of the gap (see Figure 7) are given in Figures 8 and 9. Figure 8 shows the variation of the pressure as a function of time. As can be seen, the pressure decreases from a maximum value at the initial time to its steady state value at the final time. A similar trend was experimentally measured[27].

The variation of the temperature for three
mentioned nodes are also shown in Figure 9. A gradual increment is obtained from the initial value (298 K) to the final values which are equal toward the steady state results. Comparison of the obtained results at the final time with those from the steady state ones, show the expected gradual trend of the values towards a steady state plateau. Thus the program predicts the quantitative transient behaviour of the model accurately.

CONCLUSION

We have developed a computer model based on the combination of a 2-dimensional mathematical model, a finite element solution scheme and a computer program. This computer model is used to simulate the flow of an elastomeric material inside a typical Banbury mixer under fully filled condition. Existing experimental data are in good agreement
A transient finite element model of mixing

Figure 5. Distribution of pressure in flow domain.

with model predictions. It should be pointed out that, however, the lack of a laboratory mixer which is specially equipped with highly sophisticated instrumentations to measure the model predicted parameters accurately, prevents us to check the validity of the model more exceedingly. On the other hand, there are various parameters which should be included into the described model to consider the flow and the mixing process in a real mixer. The most important of them can be expressed as the tracking of the established free surfaces, viscoelastic rheological behaviour of the rubber compounds changing in response to mixing, and slipstick of rubber on mixer walls.

We plan to continue this research project to include the mentioned parameters into our model. This will enable us to tackle the mixing problem in a more realistic condition, thus promoting the

Figure 6. Distribution of temperature in flow domain.
Figure 7. Location of the nodes used or the describing the variations of pressure and temperature with time.

agreement of results of the computer simulation to the obtained data from experiments more greatly.

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NOMENCLATURE

b Temperature sensitivity.
Cp Heat capacity.
e1,e2 Unit vectors directed along the mapped local coordinates.
F1 Load vector.
I2 Second invariant of rate-of-deformation tensor.
k Thermal conductivity.
Kst Stiffness matrix.
Mst Mass matrix.
n Power-law index.
n1,n2 Components of the unit vector normal to the boundary.
p Pressure.

Figure 8. Variation of pressure with time.

Q Generated heat due to the viscosity.
t Time.
T Temperature.
T0 Reference temperature.
u,v Components of the velocity vector.
v1,v2 Components of the velocity vector in direction of e1,e2.
w1,w2 Weight functions.
Xt Solution vector.
\gamma Rate-of-deformation tensor.
\delta A small pre-defined value used in convergence criterion.
\eta Viscosity.

Figure 9. Variation of temperature with time.
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\( \eta_0 \) Fluid consistency.
\( \theta \) Parameter in time approximation.
\( \lambda \) Penalty parameter.
\( \lambda_0 \) Relaxation time.
\( \lambda^* \) Viscosity dependent penalty parameter.
\( \lambda_e, \lambda_t \) Two parameter used in upwinding formulation.
\( \rho \) Density.
\( \tau \) Viscous stress tensor.
\( \Phi \) Upwinding parameter.
\( \psi \) Shape function.
\( \Omega \) Solution domain.

REFERENCES