

The Effect of Mechanical and Physical Properties of Polymers on Stress Amplification Factor in Composites with an Internal Damage

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ABSTRACT

The effect of mechanical and physical properties of polymer matrices on stress amplification factors is studied in a composite monolayer. The lamina is subjected to an elliptical damaged zone of radii "a" and "b" while loaded at infinity by a force "P". The matrix takes both axial and shear loads. The ratio of a/b is allowed to vary in a range of a/b, to simulate almost a crack and a circular hole, for the two values of 0 and a=b, respectively. Any reduction in stress amplification factor K_r , and peak shear stress in the matrix $(S_{xy})_{max}$, appears to well depend on the value of matrix to fibre volume fraction V_m/V_f , and their mechanical properties, as well as shape and size of the hole. For polypropylene and epoxy matrices, at a/b= 0.5 and r= 5, increasing V_m/V_f from 0.5 to 2.0 decreases $(S_{xy})_{max}$ by 62% and 73%, respectively, while K_r is decreased by 17%.

Key Words:

lamina;
damage zone;
polymer matrix;
stress amplification factor.

INTRODUCTION

The term fibre reinforced plastics (FRP) apply to any combination of a fibre embedded in a matrix which in this case is considered to be a plastic resin. The resulting composite offer many advantages over traditional materials: chief among these are

high strength, light weight, flexibility in design parts consolidation, better dimensional stability and so on. Due to the high strength and light weight of FRP, aerospace and high performance sporting goods utilize premium composites materials such

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as carbon fibres embedded in epoxy resin.

A variety of polymer matrix resins are available to engineers for fabricating FRP. The matrix or the resin, usually falls into one of the two categories; thermoset or thermoplastic. A thermoplastic resin remains solid at room temperature, while, a thermoset resin cures permanently. The trade-offs between these two types of resins have been argued extensively in terms of cost, cure time, processing temperature, and physical properties.

In fibre reinforced plastics, adding fibres to a resin matrix creates a material whose properties cannot be predicted by assuming the properties of its components [1]. In fact, one of the main advantages of composites is the complementary nature of the components. For example, thin glass fibres are quite strong but susceptible to damage. In comparison, some plastics are relatively weak but extremely versatile and tough. The combination of these components can create a material that has a better overall property than either one of the individual components. The overall property may somehow be altered in presence of an internal damage which may be caused through a variety of sources. Due to the nature of constituents in these materials, it is always assumed that most of the normal load is taken by the fibres while majority of the shear stress is sustained by the matrix. Failure of such a material may then happen through variety of ways, among which one can point to a material rupture and fibre pull out. The former is caused due to excessive normal stress in the fibres while the latter is produced by excessive shear stress in the matrix, causing a break in the existing bond with the fibres. Here, the effect of matrix mechanical properties on overall behavior must be determined so that the designer can tolerate for any change in his initial design. By definition, a lamina (or a monolayer) is a single layer of fibres embedded in a resin called matrix. The fibres may be arranged parallel to each other, as shown in Figure 1. Once these layers are stacked on top of each other, to reach a desired thickness, the resulting material is called a laminate (Figure 1). To better understand the behaviour of such a material in presence of any damage zone, a thorough micro-mechanical investigation must be performed. This zone may be created due to any error in manufacturing process or any mechanical caused human treatment (such as drilling a hole). Upon loading such a material

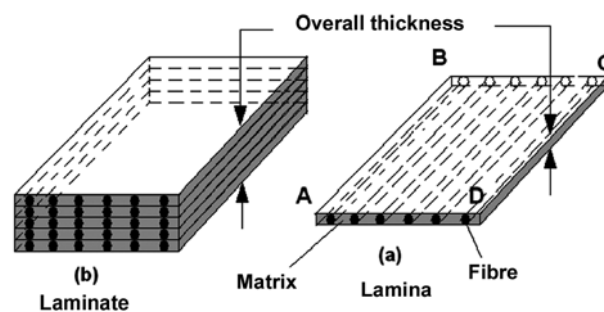


Figure 1. Fibre and matrix arrangement in a lamina and a laminate.

(a tensile load in this case) a local stress amplification K_T arises within the material, where in general, its maximum value is at the site of the imperfection. By definition, stress amplification (or stress amplification factor) is the ratio of the local load at any point, to the load applied at infinity (or far end). Obviously, this ratio is bigger than one. In this research, the local imperfection is in form of a hole where its shape is allowed to vary from that of a circle to almost a crack (not a crack). Due to stress singularity at the crack tip, the treatment of a crack becomes quite different and has been already treated. [2]. Depending on the size and shape of the imperfection, the point or the region over which this ratio reaches maximum may vary. Obviously, once the maximum value of K_T exceeds a limiting value, the damage zone may grow in size, and finally cause the material to fail.

Although polymeric resins are viscoelastic in nature, depending on their type, their behavior can be considered to be linear elastic to some extent [3,4]. For example, epoxy resins show a linear elastic behaviour up to a strain of 2% while this value can be increased to 5% in some cases [3]. A few authors have tried to apply viscoelasticity of the resin to some specific problems [5-6], but Most of the research available on polymeric matrices has treated the resin to behave as linear elastic, once it is used as a binder in FRP composites [7-9]. Test results appear to be in a good agreement with theoretical studies and this supports the validity of linear assumption made for the range of loads tested on a variety of resins such as vinyl-ester, epoxy, and polypropylene [10-14]. The stress-strain relationship for polypropylene resin has been shown to be linear elastic up to a strain of 10% [12]. The analysis has been conducted I on both polymer and metal matrix

resins[13,14]. Here, the behaviour of the metal and epoxy matrices is assumed to be elastic-perfectly plastic to account for any plastic behavior.

FORMULATION OF THE PROBLEM

In order to investigate the effect of mechanical and physical properties of polymers on stress amplification factor analytically, one has to write force equilibrium equation of any fibre and its neighbouring matrix bay and derive the necessary field equations. To do so, It is assumed that the number of fibres in the lamina is N (Figure 2). Due to the complexity and large number of equations involved in this derivation, only the intermediate steps leading toward the final field equations are introduced. In this derivation, the bond between any fibre and its neighbouring bays is assumed to be perfect. As shown in Figure 3, the lamina thickness is taken to be equal to each fibre's diameter. Both fibre and matrix are considered to behave as linear elastic upon loading.

The lamina is subjected to an internal damage zone in form of a hole while loaded axially along the fibres. In order to determine the effect of matrix properties on maximum stresses produced in the lamina in presence of this damage zone, a force equilibrium equation is written on a typical fibre of length dx in the vicinity of the hole. Using Figure 4, the result is;

$$\frac{dp_n}{dx} + h[(\tau_{xy})_{n+1,n} - (\tau_{xy} + \Delta\tau_{xy})_{n,n-1}] = 0 \quad (1)$$

Application of force equilibrium equation to a matrix bay will result into;

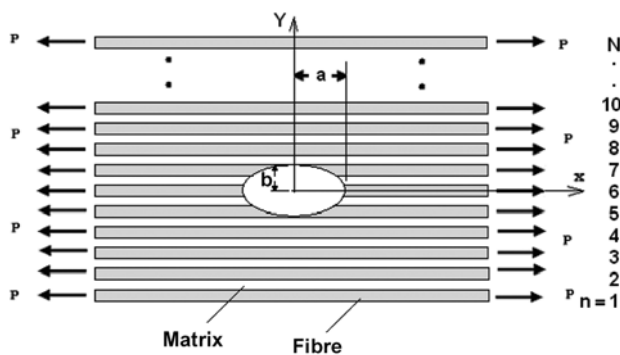


Figure 2. Fibre arrangement in a composite lamina with an internal hole.

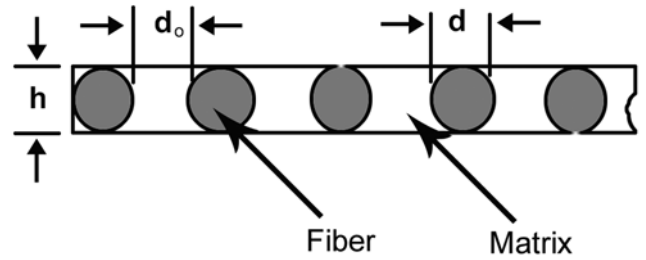


Figure 3. Cross-sections of the lamina.

$$\frac{dp_{n,n+1}^m}{dx} + h(\Delta\tau_{xy})_{n,n+1} = 0 \quad (2)$$

Here, superscript "m" stands for the matrix and $P_{n,n+1}^m$ corresponds to the load in the middle of a matrix bay bonded by the n^{th} and $(n+1)^{th}$ fibre.

The displacement equation for the matrix bay bonded by the n^{th} and $(n+1)^{th}$ fibre may be written as:

$$u_{n+1,n} = u_{n+1,n}^m + \left(\frac{u_{n+1} - u_n}{d_o}\right) y + \frac{2}{d_o^2} (u_n + u_{n+1} - 2u_{n+1,n}^m) y^2 \quad (3)$$

Whereas, shown in Figure 5, y is measured locally from center of each matrix bay.

The above equation $u_{n+1,n}^m$ represents the center displacement of each matrix bay. To find the load in a matrix bay bonded by the n^{th} and $(n+1)^{th}$ fibre, we may write;

$$P_{n+1,n}^m = \int_{-d_o/2}^{d_o/2} [E_m \frac{du_{n+1,n}}{dx}] h dy \quad (4)$$

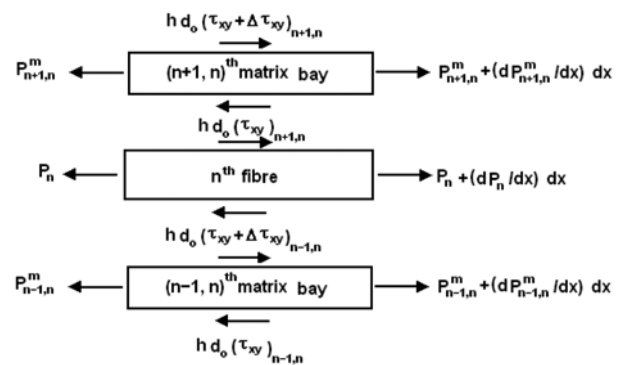


Figure 4. Force equilibrium diagram for a typical fibre and its neighbouring matrix.

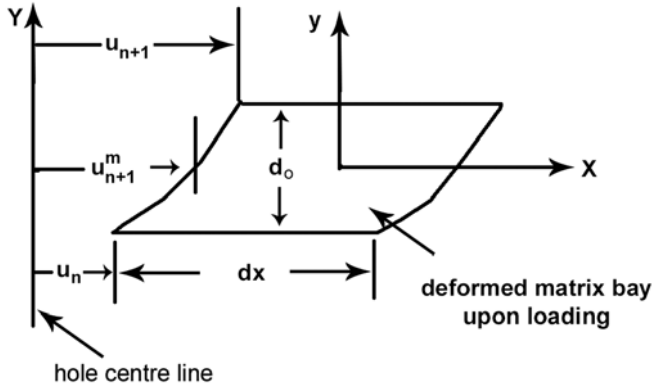


Figure 5. Deformed shape of a typical matrix bay of length dx in the vicinity of the hole.

Substituting for $u_{n+1,n}$ and integrating we have;

$$p_{n+1,n}^m = E_m A_m \left[\frac{du_{n+1,n}^m}{dx} + \frac{1}{6} \left(\frac{du_{n+1}}{dx} + \frac{du_n}{dx} - 2 \frac{du_{n+1,n}^m}{dx} \right) \right] \quad (5)$$

where it is assumed that $A_m = hd_o$. Allowing

$$e_{n+1,n} = u_{n+1} + u_n - 2u_{n+1,n}^m \quad (6)$$

then;

$$p_{n+1,n}^m = E_m A_m \left[\frac{du_{n+1,n}^m}{dx} + \frac{1}{6} \frac{de_{n+1,n}}{dx} \right] \quad (7)$$

Similarly, the shear stress in each matrix bay may be written as;

$$(\tau_{xy})_{n+1,n} = G \left\{ \frac{u_{n+1} - u_n}{d_o} + \frac{4}{d_o^2} (e_{n+1,n}) y \right\} \quad (8)$$

The above equation may be written in terms of displacement at $y = -d_o/2$ as;

$$(\tau_{xy})_{n+1,n} \Big|_{y=-d_o/2} = \frac{G}{d_o} (u_{n+1} - u_n - 2e_{n+1,n}) \quad (9)$$

Similarly, the expression for $(\Delta\tau_{xy})_{n+1,n}$ is equal to;

$$(\Delta\tau_{xy})_{n+1,n} = \int_{-d_o/2}^{d_o/2} \left(\frac{\partial \tau_{xy}}{\partial y} \right)_{n+1,n} dy = \frac{4G}{d_o} (e_{n+1,n}) \quad (10)$$

Hence, equilibrium equations (1) and (2) reduce into;

$$E_f A_f \frac{d^2 u_n}{dx^2} + \frac{Gh}{d_o} (u_{n+1} - 2u_n + u_{n-1}) - \frac{2Gh}{d_o} (e_{n+1,n} + e_{n,n-1}) = 0 \quad 1 < n < N \quad (11)$$

$$E_m A_m \left[\frac{d^2 u_{n+1,n}^m}{dx^2} + \frac{1}{6} \frac{d^2 e_{n+1,n}}{dx^2} \right] + \quad (12)$$

$$\frac{4Gh}{d_o} (e_{n+1,n}) = 0 \quad 1 < n < N$$

Using equation (6), the above equation may be re-written as;

$$\frac{E_m A_m}{2} \left[\frac{d^2 u_n}{dx^2} + \frac{d^2 u_{n-1}}{dx^2} - \frac{2}{3} \frac{d^2 e_{n,n-1}}{dx^2} \right] + \frac{4Gh}{d} (e_{n,n-1}) = 0 \quad 1 < n < N \quad (13)$$

For the two fibres bonding the lamina's edges, the equilibrium equations reduce into:

$$E_f A_f \frac{d^2 u_n}{dx^2} + \frac{Gh}{d_o} (u_{n-1} - u_n) - \frac{2Gh}{d_o} (e_{n,n-1}) = 0 \quad n = N \quad (14)$$

$$E_f A_f \frac{d^2 u_n}{dx^2} + \frac{Gh}{d_o} (u_{n+1} - u_n) - \frac{2Gh}{d_o} (e_{n+1,n}) = 0 \quad n = 1 \quad (15)$$

NON-DIMENSIONAL PARAMETERS

For simplicity, the equilibrium eqn (11), and eqns (13)-(15) may be written in non-dimensional form as follows:

$$x = \sqrt{\frac{E_f A_f d}{Gh}} \xi \quad (16)$$

$$u_n = p \sqrt{\frac{d}{E_f A_f Gh}} U_n$$

$$u_{n,n-1}^m = p \sqrt{\frac{d}{E_f A_f Gh}} U_{n,n-1}^m$$

$$e_{n,n-1} = p \sqrt{\frac{d}{E_f A_f Gh}} E_{n,n-1}$$

$$p_n = p P_n$$

$$k_r = (P_n)_{\max} = \frac{(p_n)_{\max}}{p}$$

$$p_{n,n-1}^m = p P_{n,n-1}^m$$

$$\tau_{xy} = p \sqrt{\frac{G}{E_f A_f}} S_{xy} \text{hd}$$

$$\delta = \sqrt{\frac{Ghd}{E_f A_f}}$$

$$\varepsilon = \frac{1}{2} \left[\frac{A_m E_m}{A_f E_f} \right] = \frac{1}{2} \left[\eta \frac{E_m}{E_f} \right]$$

$$\frac{1}{\eta} = \frac{d_o}{d} = \frac{V_m}{V_f}$$

All the above parameters are defined in (nomenclature/Selection). By definition, V_f and V_m are fibre and matrix volume fractions in the lamina, respectively.

Taking $A_f = \text{hd}$ and $A_m = \text{hd}_o$, then one can show that;

$$\frac{V_m}{V_f} = \frac{(N-1)}{N} \eta \quad (16b)$$

Using the above parameters, the equilibrium equations may now be re-written as;

$$\frac{d^2 U_n}{d\xi^2} + \eta(U_{n+1} - 2U_n + U_{n-1}) - 2\eta E_{n+1,n} - 2\eta E_{n,n-1} = 0, \quad 1 < n < N \quad (17)$$

$$\varepsilon \left[\frac{d^2 U_n}{d\xi^2} + \frac{d^2 U_{n-1}}{d\xi^2} - \frac{2}{3} \frac{d^2 E_{n,n-1}}{d\xi^2} \right] + 4\eta E_{n,n-1} = 0, \quad 1 < n < N \quad (18)$$

$$\frac{d^2 U_n}{d\xi^2} + \eta(U_{n-1} - U_n) - 2\eta E_{n,n-1} = 0 \quad (19)$$

$$\frac{d^2 U_n}{d\xi^2} + \eta(U_{n+1} - U_n) - 2\mu E_{n+1,n} = 0 \quad (20)$$

In terms of displacements, these equations may be written as;

$$\frac{d^2 U_n}{d\xi^2} + \eta(-U_{n-1} - 6U_n - U_{n+1}) + 4\eta U_{n+1,n} + 4\eta U_{n,n-1}^m = 0 \quad 1 < n < N \quad (21)$$

$$\frac{\varepsilon}{3} \left[\frac{d^2 U_n}{d\xi^2} + \frac{d^2 U_{n-1}}{d\xi^2} + \frac{4}{3} \frac{d^2 U_{n,n-1}^m}{d\xi^2} \right] + 4\eta(U_n + U_{n-1} - 2U_{n,n-1}^m) = 0 \quad 1 < n < N \quad (22)$$

$$\frac{d^2 U_n}{d\xi^2} + \eta(-3U_n - U_{n-1}) + 4\eta U_{n,n-1}^m = 0 \quad n = N \quad (23)$$

$$\frac{d^2 U_n}{d\xi^2} + \eta(-3U_n - U_{n+1}) + 4\eta U_{n+1,n}^m = 0 \quad n = 1 \quad (24)$$

DISPLACEMENT AND LOAD FIELDS

The above system of differential-difference equations may now be written in a matrix notation as:

$$L_1 U'' - L_2 U = 0 \quad (25)$$

where;

$$[U]^T = [U_q U_{q,q-1}^m U_{q-1} \dots U_{-q+1} U_{-q,-q+1}^m U_{-q}] \quad (26)$$

$$[U^{11}]^T = [U_q'' U_{q,q-1}^{n_m} U_q'' \dots U_{-q+1}'' U_{-q,-q+1}^{n_m} U_q''] \quad (27)$$

In the set of eqns (25), L_1 and L_2 are coefficient matrices. Assuming a solution of the form $U = \text{Re}^{\lambda\xi}$ for displacements, and substituting this expression back into eqn (25), we have;

$$(L_2 - \lambda^2 L_1) R = 0 \quad (28)$$

Upon determination of the eigen values of the above equations, the solution to the set of equations (25) may be written as;

$$U_n = \xi + \sum_{i=1}^{2N-1} C_i R_{2n-1}^{(i)} e^{\lambda_i \xi} \quad 1 \leq n \leq N \quad (29)$$

$$U_{n,n-1}^m = \xi + \sum_{i=1}^{2N-1} C_i R_{2n-2}^{(i)} e^{\lambda_i \xi} \quad 1 \leq n \leq N \quad (30)$$

$$P_n = \frac{\partial U_n}{\partial \xi} = 1 + \sum_{i=1}^{2N-1} C_i \lambda_i R_{2n-1}^{(i)} e^{\lambda_i \xi} \quad 1 \leq n \leq N \quad (31)$$

$$P_{n,n-1}^m = 2 \varepsilon \left\{ 1 + \sum_{i=1}^{2N-1} \left[\frac{2}{3} R_{2n-2}^{(i)} + \frac{1}{6} [R_{2n-1}^{(i)} + R_{2n-3}^{(i)}] C_i \lambda_i e^{\lambda_i \xi} \right] \right\}, \quad 2 \leq n \leq N \quad (32)$$

In above equations, λ_i correspond to eigen values of equation (28), and an expression such as $R_{2n-1}^{(i)}$ corresponds to a value associated with the $(2n-1)^{th}$ row of the i^{th} eigen vector. All positive values of λ_i are discarded due to the bondness condition on load and displacement of each fibre and matrix bay as $\xi \rightarrow \infty$.

BOUNDARY and Bondness Conditions

To solve for the unknowns values of C_i , due to symmetry, the following boundary and bondness conditions may be applied to all intact filaments and matrix bays.

$$U_n(0) = 0 \quad \text{in all intact fibres} \quad (33)$$

$$U_{n,n+1}^m(0) = 0 \quad \text{in all intact matrix bays} \quad (34)$$

Using Figure 6, the force balance on a typical cut fibre and its neighbouring bay at the hole's free surface, will result into;

$$p_{n+1,n}^m + hd_0 \left(\tau_{xy} \Big|_{-\frac{d_0}{2}} + \Delta \tau_{xy} \right)_{n+1,n} \tan \theta_1 = 0 \quad (35)$$

$$p_n + hd_0 \left(\tau_{xy} \right)_{n+1,n} \Big|_{-\frac{d_0}{2}} \tan \theta_2 = 0 \quad (36)$$

Substituting the expressions for shear stresses in terms of displacements we have;

$$\left(\frac{\delta}{6} \left[\frac{dU_n}{d\xi} + \frac{dU_{n+1}}{d\xi} + 4 \frac{dU_{n+1,n}^m}{d\xi} \right] + \frac{G}{E_m \eta} [U_n + 3U_{n+1} - 4U_{n+1,n}^m] \tan \theta_1 \right) \Big|_{\xi=\xi_n} = 0 \quad (37)$$

$$\left(\delta \frac{dU_n}{d\xi} + \frac{G}{E_f} (-3U_n - U_{n+1} + 4U_{n+1,n}^m) \right) \tan \theta_2 \Big|_{\xi=\xi_n} = 0 \quad (38)$$

Equations (33)-(34) and (35)-(36) may now be written in terms of unknowns as;

$$\sum_{i=1}^{2N-1} C_i R_{2n-1}^{(i)} = 0 \quad \text{for all intact fibres} \quad (39)$$

$$\sum_{i=1}^{2N-1} C_i R_{2n-2}^{(i)} = 0 \quad \text{for all intact matrix bays} \quad (40)$$

And for each cut fibre and its neighboring matrix bays we have;

$$\begin{aligned} & \frac{\delta}{6} \left[6 + \sum_{i=1}^{2N-1} C_i R_{2n-1}^{(i)} \lambda_i e^{\lambda_i \xi_n} + \sum_{i=1}^{2N-1} C_i R_{2n+1}^{(i)} \lambda_i e^{\lambda_i \xi_{n+1}} + 4 \sum_{i=1}^{2N-1} C_i R_{2n}^{(i)} \lambda_i e^{\lambda_i \xi_{n+1,n}} \right] \\ & + \frac{G}{\eta E_m} [\xi_n + \sum_{i=1}^{2N-1} C_i R_{2n-1}^{(i)} e^{\lambda_i \xi_n} + 3\xi_{n+1} + 3 \sum_{i=1}^{2N-1} C_i R_{2n+1}^{(i)} e^{\lambda_i \xi_{n+1}} - 4\xi_{n+1,n}] \end{aligned}$$

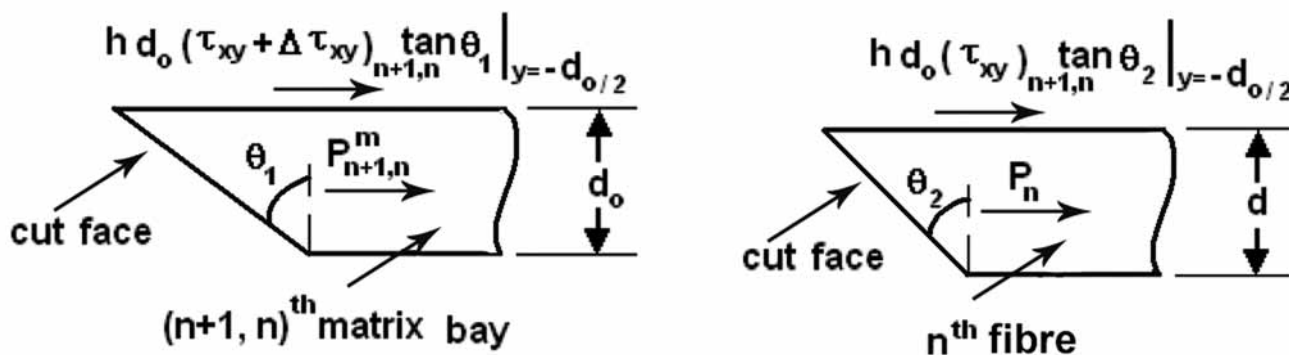


Figure 6. Force balance on a typical cut fibre and its neighbouring matrix bay.

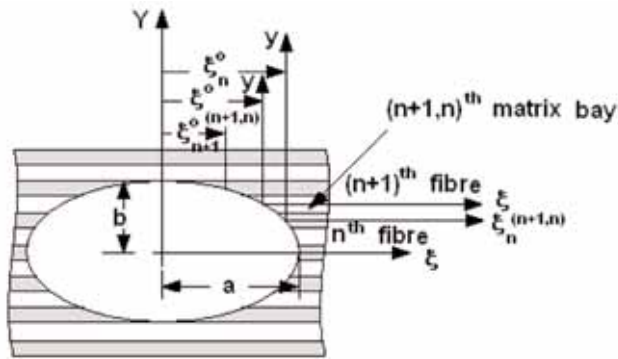


Figure 7. Local coordinate system representation for cut fibres.

$$-4 \sum_{i=1}^{2N-1} C_i R_{2n}^{(i)} e^{\lambda_i \xi_{n+1,n}} \tan \theta_1 = 0 \quad (41)$$

$$M + t \leq n < M + 2t$$

$$\delta \left[1 + \sum_{i=1}^{2N-1} C_i R_{2n-1}^{(i)} \lambda_i e^{\lambda_i \xi_n} \right] +$$

$$\frac{G}{E_f} \left[-3 \xi_n - 3 \sum_{i=1}^{2N-1} C_i R_{2n-1}^{(i)} e^{\lambda_i \xi_n} - \xi_{n+1} \right]$$

$$- \sum_{i=1}^{2N-1} C_i R_{2n+1}^{(i)} e^{\lambda_i \xi_{n+1}} +$$

$$4 \xi_{n+1,n} + 4 \sum_{i=1}^{2N-1} C_i R_{2n}^{(i)} e^{\lambda_i \xi_{n+1,n}} \tan \theta_2 = 0 \quad (42)$$

$$M + t \leq n \leq M + 2t$$

Where $t=(r-1)/2$.

Similar expressions may be written for each fibre and its matrix bays at the cut site of the lamina, in the bottom half portion of the hole, where "Y" is negative (Figure 7). In eqns (41) and (42), the parameters ξ_n and $\xi_{n+1,n}$ refer to the values of ξ at the free surface of each fibre and its neighbouring matrix bay, cut by the hole. According to Figure 7, a parameter such as ξ_n may be defined as:

$$\xi - \xi_n^0 = \xi_n \quad (43)$$

MECHANICAL PROPERTIES

In order to investigate the effect of matrix mechanical properties on maximum values of shear stress $(S_{xy})_{\max}$ and K_r , in presence of an internal damage, using refs. [3] and [12], the following data were selected for each

Table 1. Mechanical properties of selected matrix and fibres.

Element	Elastic modulus (GPa)
Epoxy resin	2.8-4.2
Nylon 6 resin	3.4
Polypropylene resin	1.3
Glass fibre	
C-Glass	69
E-Glass	70 - 72.4
S-Glass	85.5

fibre and matrix bay (Table 1). With the aid of the above data and direct substitution of boundary and bondness conditions given in the previous section, equilibrium equations are solved and the required results were obtained.

RESULTS AND DISCUSSION

In order to investigate the effect of polymer matrix properties on stress amplification factor in the lamina, the elastic modulus of the matrix was taken to vary from 1.3 GPA (for polypropylene) to 3.8 GPA (3.45 GPA for epoxy resins and 3.4 GPA for nylon 6). The matrix is assumed to be stiffened by glass fibres with a modulus of elasticity equal to 70 GPA. Furthermore, to keep the hole symmetric with respect to x-axis (Figure 2), it is assumed odd number of fibres are cut by the hole. The ratio of hole's larger diameter a to smaller diameter b was allowed to vary from 1.0 to 0.10, simulating a circular hole and a sharp imperfection, respectively. The sharp imperfection tends to act as almost a crack once a approaches zero. For a complete crack, parameter a is equal to zero ($a/b=0$). The total number of fibres is allowed to be 21. The parameter K_r presented in Figures 8-10 is the peak value of normal stress amplification factor in the lamina which happens to be in the first intact filament bonding the damage zone at $x = 0$. The peak value of shear stress presented in Figures 11 and 12 happens to be in the bay bonded by the last broken fibre and the first intact filament. As stated earlier, excessive values of K_r or $(S_{xy})_{\max}$ can cause a failure in the lamina in terms of material rupture or fibre pull out.

To investigate the accuracy of the results in this

Table 2. Compositions and some properties of OBD.

Number of broken fibres "r"	Values of K_r in present work				Values of K_r in ref. [2] (crack, a=0)
	a/b = 0.5	a/b = 0.25	a/b = 0.1	a/b = 0.02(approaching a crack)	
3	1.48	1.53	1.57	1.59	1.65
5	1.71	1.82	1.89	1.94	2.0
7	1.95	2.09	2.19	2.28	2.35

research, values of K_r from the present work were compared with those of Ref. [2] for a crack. Here, the a/b ratio was allowed to approach zero, to simulate the results of a crack. The comparison is for $V_m/V_f = 1.0$, since this is the only value used in above reference. The results are shown in Table 2. Here "r" designates the number of broken fibres which in turn controls the size of the crack in ref. [2], and the shape and size of the hole in present work. As realized for a/b approaching zero, values of K_r approach those of a crack.

Figure 8 shows the effect of imperfect shape and size on K_r for different values of V_m/V_f ratio. Here it is assumed the glass fibres are embedded in epoxy matrix. It is important to note that holding r as a constant, any change in a/b ratio causes a change in the shape (not the size) of the hole. For example, at r = 5, a/b = 1 corresponds to a circular hole while $0 < a/b < 1$ represents an elliptical hole with its major diameter normal to the fibres. As a/b ratio approaches zero, the hole becomes sharper and its shape reaches that of almost a crack (for a crack a/b = 0). The values of V_m/V_f ratio were allowed to vary from 0.5 to 3.5. For any specific value of a/b, an increase in volume fraction ratio causes a decrease in stress amplification factor K_r . For example,

at a/b = 1, and r = 5, once the ratio of V_m/V_f is increased from 0.5 to 3.5, the value of stress amplification factor is reduced from 1.68 to 1.27. This corresponds to a 24.4% decrease in the value of K_r . Similar reductions were observed for other values of a/b. For example, for values of a/b = 0.5 and 0.1, the percentage decrease in K_r is 27%.

Since in composite materials with polymeric matrix almost all the applied normal load is taken by the fibres (matrix is weak in tension), and furthermore, volume fraction of the individual constituents can be easily controlled in different designs, then lowering the value K_r would lower the value of normal stress in the fibres. This in turn would reduce the risk of any damage growth and final rupture of the material.

In Figures 8 and 9, it is assumed that the damage zone has cut five and nine fibres, respectively. This means the hole considered in Figure 9 is 1.5 times larger than that considered in Figure 8. According to figure 9, for a/b = 1.0, as the ratio of V_m/V_f increases from 0.5 to 3.5, the value of K_r reduces by 36.9%. For a/b = 0.1, this reduction seems to be 35.1%. Comparing the results of this figure with those in Figure 8, one can conclude that the effect of an increase in V_m/V_f ratio on any reduc-

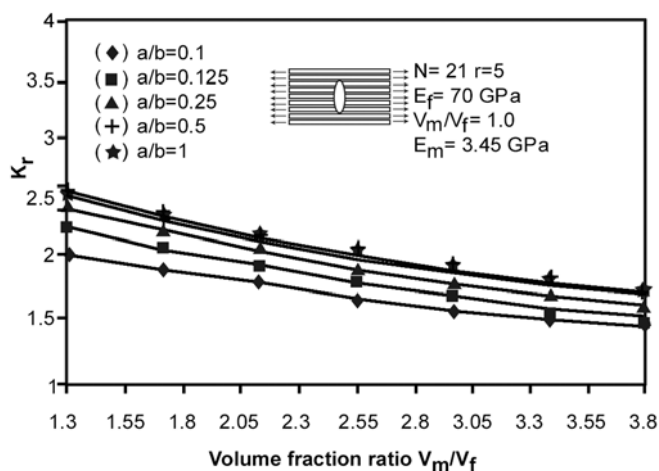


Figure 8. Variation of stress amplification factor in the lamina vs. volume fraction for various types of imperfections.

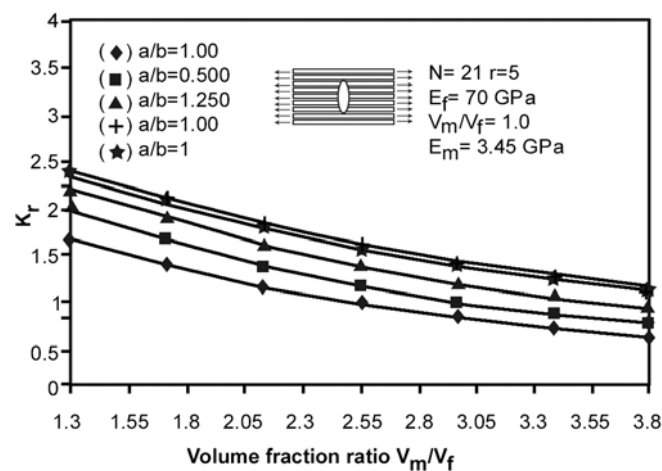


Figure 9. Variation of stress amplification factor in the lamina vs. volume fraction for various types of imperfections.

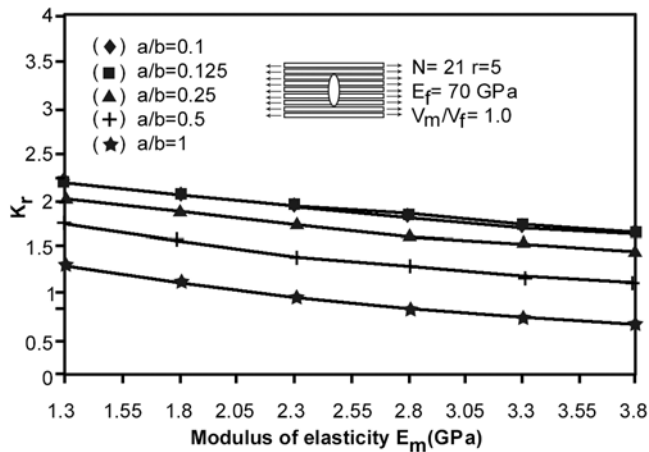


Figure 10. The effect of matrix elastic modulus variation on normal stress amplification factor

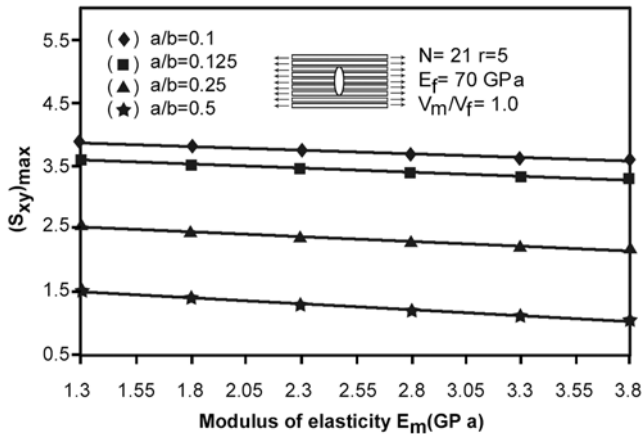


Figure 11. The effect of matrix elastic modulus variation on maximum shear stress in the lamina

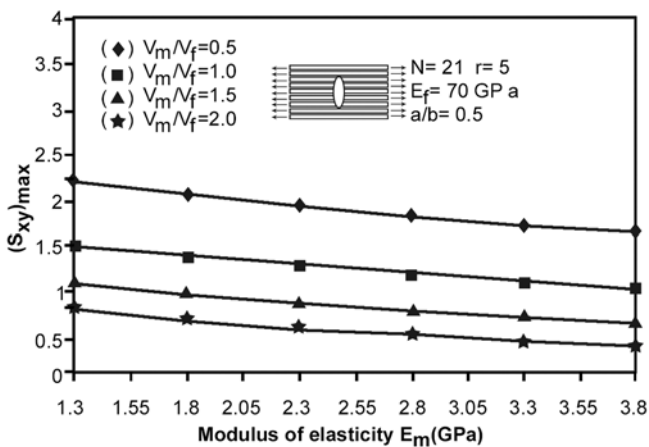


Figure 12. The effect of matrix elastic modulus variation on maximum shear stress in the lamina.

tion in K_t becomes more pronounced for larger holes.

Figure 10 shows the effect of matrix elastic modulus on stress amplification factor K_t . The results are for $r = 5$, $V_m/V_f = 1.0$ and values of a/b ranging from 0.1 to 1.0. According to this figure, an increase in E_m from 1.3 GPa (polypropylene) to 3.45 GPa (epoxy), can cause a 6.0% decrease in K_t at $a/b = 0.1$ and 9% decrease at $a/b = 1.0$. Hence, in composites with polymeric resins, although the matrix is weak in tension, any small increase in E_m from 1.3 GPa to 3.34 GPa can cause a noticeable reduction in K_t .

The effect of matrix elastic modulus E_m on peak shear stress developed in the matrix is examined in Figures 11 and 12. As previously stated, the location of this stress happens to be in the last matrix bay cut by the damage zone, and adjacent to the first intact filament. According to Figure 11, for any specific value of E_m , as the shape of the imperfection approaches that of a circle, the value of $(S_{xy})_{max}$ decreases considerably. Furthermore, for any constant value of a/b ratio, any increase in E_m will reduce the value of $(S_{xy})_{max}$ considerably. According to this figure, an increase in E_m from 1.3 GPa to 3.8 GPa, will reduce the peak shear stress by 30.7% at $a/b = 0.5$ and 7.5% at $a/b = 0.1$. Hence, the effect of E_m on $(S_{xy})_{max}$ becomes more pronounced for a more rounded imperfection.

To further investigate the effect of volume fraction ratio on peak shear stress, values of $(S_{xy})_{max}$ were calculated and plotted as a function of E_m as shown in Figure 12. As it appears, for any value of E_m , an increase in V_m/V_f , decreases the peak shear stress in the matrix considerably. For example, for $E_m = 1.3$ GPa, the value of $(S_{xy})_{max}$ has been reduced from 2.21 at $V_m/V_f = 0.5$ to 0.83 at $V_m/V_f = 2.0$. This corresponds to a 62% reduction in $(S_{xy})_{max}$. This behaviour is observed for other values of E_m . Obviously, any decrease in $(S_{xy})_{max}$ can help to lower the risk of any bond failure between the fibres and their neighbouring matrix bays. As a result, this would lower the risk of any fibre pull out which is considered to be a type of material failure.

CONCLUSION

The effect of physical and mechanical properties of polymeric matrices on stress resultants has been stud-

ied analytically in a composite lamina subjected to an internal damage. The results are tailored to include the effect of matrix volume fraction V_m (in terms of V_m/V_f ratio), elastic modulus of the matrix E_m , shape and size of the imperfection, on overall behaviour of the lamina. The stress resultants are presented in terms K_r and the peak shear stress in the matrix $(S_{xy})_{max}$. According to the results, for glass fibres embedded in epoxy resin, an increase in V_m/V_f from 0.5 to 3.5 (increase in volume content of the matrix) causes a 24% reduction in peak normal stress in the fibres at $a/b = 1.0$ (circular hole with $r = 5$). This means increasing the volume content of the matrix can lower the risk of material rupture considerably. For larger holes, this effect is more pronounced. For a constant hole size (say $r = 5$), and for any specific value of V_m/V_f , the value of K_r increases as the ratio of a/b becomes smaller (from a circular hole toward almost a crack). This means that the geometric shape of the hole has a significant effect on the value of K_r and hence, final rupture of the composite lamina. Holding V_m/V_f as a constant, an increase in elastic modulus of the matrix can decrease the value of K_r . The amount of decrease is 9% as E_m is increased from 1.3 (polypropylene resin) to 3.45 GPa (epoxy resin) at $V_m/V_f = 1.0$. It is also recognized that, an increase in V_m/V_f ratio or E_m , can decrease the peak shear stress in the matrix considerably. For polypropylene and epoxy matrices, the reductions appear to be 62% and 73%, respectively, at $a/b = 0.5$, and $r = 5$. This means that increasing the volume content of the matrix (or its elastic modulus), can lower considerably the risk of any bond failure, and hence, fibre pull out. Holding V_m/V_f and E_m as a constant, any change in shape of the imperfection in terms of an increase in a/b ratio, will lower the value of peak shear stress in the matrix considerably. The amount of reduction appears to be 61% for epoxy matrix and 70% for polypropylene, provided at $V_m/V_f = 1.0$ and $r = 5$, a/b ratio increases from 0.1 to 0.5. Hence any reduction in peak shear stress in the matrix becomes mostly sensitive to V_m/V_f ratio as well as shape and size of the hole.

NOMENCLATURE

A_f, A_m	Cross-sectional areas of the fibres and each matrix bay, respectively.
a	Larger diameter of the elliptic hole.

b	Smaller diameter of the elliptic hole.
d	Fibre diameter
d_o	Fibre spacing
E_f, E_m	Fibre and matrix elastic moduli, respectively
G	Shear modulus of the matrix
h	Thickness of the lamina
K_r	Stress amplification factor in each fibre
N	Total number of fibres
p	Normal load applied to the lamina at infinity
p^m	Resultant normal load in the matrix
p_n	Local normal load in the n^{th} fibre
$(p_n)_{max}$	Maximum local load in the fibre (which is in the first intact filament bonding the hole)
r	Total number of broken fibres in each case
S_{xy}	Non-dimensional shear stress in each matrix bay
u	Fibre displacement
$u^m_{(n+1),n}$	Centre Displacement of the matrix bay bonded by the n^{th} and $(n+1)^{\text{th}}$ fibre
V_f	Fibre volume fraction
V_m	Matrix volume fraction
x, y	Coordinate system (Figures 2 and 5)
η	Ratio of fibre spacing to fibre's diameter
ξ	Non-dimensional coordinate along any fibre
ξ_n	Non-dimensional local coordinate along the n^{th} fibre
ξ_n^o	Non-dimensional distance from centre of the hole up to cut site for the n^{th} fibre

REFERENCES

1. Tang B., Fibre reinforced polymer composites application in USA, The First Korea/USA Workshop Proc., Jan. 28-29, 1-11 (1997).
2. Shishehsaz M., The effect of matrix extension on fibre stresses and matrix debonding in a hybrid composite monolayer, *Iran. J. Sci. Technol.*, Transaction B., **25**, 253-264 (2001).
3. Lubin G., *Handbook of Composite Materials*, Van Nostrand Reinhold Co., p. 140 (1982).
4. Shishehsaz M.R., Donatelli A.A., Tensile properties of polyethylene blends, *Polym. Eng. Sci.*, **21**, 869-872 (1981).
5. Beyerlein I.J., Stress fields around cracks with a viscous matrix and discontinuous fibre bridging, *Comp. Sci. Technol.*, **60**, 2309-2326 (2000).

6. Allam M.N.M., Zenkour A.M., Stress concentration factor of a structurally anisotropic composite plate weakened by an oval opening, *Comp. Struc.*, **61**, 199-211 (2003).
7. Reedy E.D. Jr., Fibre stresses in a cracked monolayer: Comparison of shear-lag and 3-D finite element predictions, *J. Comp. Mater.*, **18**, 595- (Nov. 1984).
8. Wang Y., Gowayed Y., Kong X., Li J., and Zhao D. Properties and analysis of composite reinforced with E-glass weft-knitted fabrics, *J. Comp. Technol. Res.*, **17**, 283-288 (1995).
9. Wang, Y., Li. J., Do P.B., Properties of composite laminates reinforced with knitted multi-layer non-crimp fabrics, *J. Comp. Mater.*, **29**, 2317-2333 (1994).
10. Ko W.L., Nagy A., Francis P.H., Lindholm U.S., Crack extension in filamentary materials, *Eng. Frac. Mech.*, **8**, 411-418 (1976).
11. Wu T.J., Hahn H.T., The bearing strength of E-glass/vinyl-ester composites fabricated by VARTM, *Comp. Sci. Technol.*, **58**, 1519-1529 (1998).
12. Fu S.-Y., Lauke B., Mäder E., Yue C.-Y., Hu X., Tensile properties of short-glass-fibre and short-carbon-fibre reinforced polypropylene composites, *J. Comp., Part A: Appl. Sci. Manuf.*, **31**, 1117-1125 (2000).
13. Xia Z., Okabe T., Curtin W.A., Shear-lag versus finite element methods for stress transfer in fibre-reinforced composites, *J. Comp. Sci. Technol*, **62**, 1141-1149 (2002).
14. Sirivedin S., Fenner D.N., Nath R.B., Galiotis, C., Effects of inter-fibre spacing and matrix cracks on stress amplification factors in carbon-fibre/epoxy matrix composites. Part I: planar array of fibres, *J. Comp., Part A: Appl. Sci. Manuf.*, **34**, 1227-1234 (2003).