A Laser Doppler Anemometry Study of the Rushton Turbine Velocity Profile for Mixing of Polymeric Liquids

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Abstract

Velocity profiles are helpful for the confident design of mixing tanks in the transition region. In this work, mean and fluctuating tangential and radial velocity components were measured using a two-component laser doppler anemometry (LDA) for a typical Rushton turbine impeller. The working fluids were different concentrations of polyacrylamide (PAA) solutions with rheological properties typical of those found in polymeric processes. It is shown that the correlations for mean and fluctuating velocities of Newtonian and inelastic non-Newtonian fluids do not apply to the case of polymeric viscoelastic liquids. New correlations are given in the lower part of the transition region (i.e., 35<Re<1800) for mean tangential and radial velocity components and also corresponding fluctuation values along the discharge plane of the impeller. It is also shown that for a given concentration of PAA solution, the dimensionless mean (radial and tangential) velocity distributions are nearly independent of impeller speed.

Key Words:
laser doppler anemometer; Rushton turbine impeller; polymeric liquids; stirred tank; velocity profile.

INTRODUCTION

Polymer processing in stirred tank reactors has received significant interest in chemical industries. One of the main objectives of polymer engineering research is to increase the predictive capabilities with respect to the effect of process variables [1]. Although it is now recognized that close clearance impellers are more effective for mixing rheologically complex fluids, however, the classical Rushton turbine remains the most common impeller in industrial and research equipment.
particularly in the fermentation industry [2]. The latter impeller induces a strong radial discharge stream and variations in the radial velocity component with impeller blade angle [3].

For many industrial materials, such as viscoelastic polymeric liquids, suspensions, etc., it is impossible or impractical to operate under turbulent mixing. One then has to proceed in the laminar or at best in the transition region. Due to the complexities and uncertainties of mixing in the transition region, it is difficult to predict mixing performance and power requirement for rheologically complex non-Newtonian fluids [4].

The influence of the complex non-Newtonian properties (in particular, viscoelasticity), secondary flow patterns and flow irregularities are expected to be more and more pronounced as the inertial forces become more and more important. This area of research is still in an early exploring stage although some works have been carried out in this field, e.g., Cheng and Carreau [5].

Different computational techniques for accurate and low cost design methods have been emphasized for impellers [6]. These methods such as the finite difference or finite element may be implemented for agitated vessels. However, detailed methods are required to validate the calculations. Laser doppler velocimetry has proven to be more accurate for the measurement of flow fields in stirred tanks than other techniques such as Pitot probes and hot-wire anemometers because:

(a) it provides flow information even in unsteady and highly turbulent flow regions as well as in the return flow areas of the tank.

(b) it operates without disturbing the flow field [6].

The first application of Laser Doppler Anemometry (LDA) to mixing tanks stirred by a Rushton turbine was by Reed et al. [7] while, the most extensive application was by Van der Molen and Van Maanen [8] who found that in the turbulent region, the dimensionless mean radial velocity in the vicinity of impeller tip and on its centerline (discharge plane) was as following:

$$\frac{\nu_r}{\nu_{tip}} = 0.85 \xi (r/R)^{-7/6}$$  \hspace{1cm} (1)

This relationship was stated as independent of the ratio of impeller to vessel diameter up to the value of 0.5 [10].

Koutsakos et al. [11] in an LDA study of inelastic non-Newtonian fluids in agitated vessels showed that the mean radial velocity profile at discharge plane of Rushton turbine impeller is:

$$\frac{\nu_r}{\nu_{tip}} = 0.85\xi (r/R)^{-7/6}$$  \hspace{1cm} (2)

where $\xi$ was introduced in their study as a measure of deviation from full turbulence of the well mixed condition described by eqn (1). Their results were described by the following condtitinos:

for $Re_a < 60$ \hspace{1cm} $\xi \propto Re_a$ \hspace{1cm} (3a)

for $60 < Re_a < 10^4$ \hspace{1cm} $\xi \propto Re_a^{0.2}$ \hspace{1cm} (3b)

for $Re_a > 10^4$ \hspace{1cm} $\xi = 1$ \hspace{1cm} (3c)

Dyster et al. [12] recently reviewed by Mavros et al. [13] and Sch fer et al. [6] gave a similar expression for radial velocity profiles for Newtonian fluids at discharge plane of Rushton turbine impeller as eqn (2), however, the parameter $\xi$ was introduced in their study as:

for $Re < 20$ \hspace{1cm} $\xi \propto Re$ \hspace{1cm} (4a)

for $Re < \sim 500$ \hspace{1cm} $\xi = 1$ \hspace{1cm} (4b)

for $Re \approx 10^4$ \hspace{1cm} $\xi = 1$ \hspace{1cm} (4c)

for $Re < \sim 500$, the parameter $\xi$ starts to fall and at $Re < 20$, gives values as low as 0.07.

There are a few studies on the determination of tangential velocity profile for polymeric fluids at discharge plane of Rushton turbine impeller in mixing tanks. On the other hand, most of them are concerned with measurements of velocity profiles in water representing the agitation of low viscous inelastic fluids. Thus, there is a lack of experimental data for the velocity profiles of polymeric fluids in mixing processes. The objective of this study is to investigate mean velocity profiles (tangential and radial) in mixing of polymeric viscoelastic liquids. In this work, different concentrations of PAA solutions are considered as representative. Besides, the effect of concentration of PAA solutions and impeller speeds are studied.
Experimental
Measurements were performed in a cylindrical tank of Plexiglas with an inside diameter of 0.276 m and wall thickness of 0.003 m. The tank included four equally spaced baffles of width of 0.03 m. The height of liquid was 0.188 m. The impeller was a typical Rushton turbine with diameter of 0.104 m. The tank and turbine configurations are shown in Figure 1. The impeller was driven by a variable speed electric motor. The entire tank and motor assembly was mounted on an automated-controlled traversing mechanism allowing the user to conduct a complete scan of the mixing tank.

The LDA system (Dantec Measurement Technology) was operated in the back scatter mode with both receiving and transmitting optics in the same module. The system consisted of a 5W Spectra-Physics argon-ion laser, two-colour 55x modular optics, two burst spectrum analyzers and a PC.

The front focusing lens had a focal length of 0.31 m and produced a beam angle of 9.92°. Additional equipment included an automated three-axis traversing system. The power of the emitted beam (blue-green) could be regulated up to 5 W. This beam was split by a modular optical system into four beams in such a way that two of them were blue rays (488.0 nm) and two others were green (514.5 nm). Radial and tangential velocities could be obtained simultaneously when the probe traversed along the radius of the mixing vessel. Tangential velocities were obtained through the use of green beams and radial velocities by using blues.

Data acquisition and processing of the particle velocity information were done using two Dantec burst spectrum analyzers (BSA). The natural ingredients of PAA solutions were adequate for LDA since for a total of 20k bursts, data rates up to over 1 kHz were easily obtained with more than 60% validity and therefore, no seeding was required [14]. The total number of collected bursts at each point was such that for any higher number of bursts, only variations less than 0.01 m/s in velocity could be sensed.

An oversize filter accepted only the signals from the smallest particles. Calculated average values could be biased in such flows. Continuous-wave mode of measurement took more data from slower-moving particles and in this way reduced the bias error to less than 2 percent [15].

Several concentrations of PAA (Magnofloc LT27 polyacrylamide) solutions in 50/50 (wt%) mixtures of glycerin/water were used as typical viscoelastic polymeric liquids (Table 1). Steady and oscillatory shear experiments were performed by means of a Haake rheometer, RV100/CV100, fully controlled by computer. Rheometric data were measured at the same temperature as that encountered in the mixing experiments. Some useful dimensionless groups characterizing vis-

![Figure 1. Set up of LDA and agitated vessel for mixing of polymeric liquids.](image-url)

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Conc. (ppm)</th>
<th>ρ (kg/m³)</th>
<th>η₀ (kg/ms)</th>
<th>a₀</th>
<th>b₀</th>
<th>c</th>
<th>d</th>
<th>λ₀ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>500</td>
<td>1114</td>
<td>0.011</td>
<td>-0.614</td>
<td>-0.179</td>
<td>-1.621</td>
<td>0.98</td>
<td>3.225</td>
</tr>
<tr>
<td>b</td>
<td>750</td>
<td>1114.8</td>
<td>0.052</td>
<td>0.0556</td>
<td>0.398</td>
<td>-2.14</td>
<td>0.097</td>
<td>3.817</td>
</tr>
<tr>
<td>c</td>
<td>900</td>
<td>1120.4</td>
<td>0.0532</td>
<td>0.0679</td>
<td>0.344</td>
<td>-2.383</td>
<td>0.0985</td>
<td>4.97</td>
</tr>
<tr>
<td>d</td>
<td>1350</td>
<td>1128.8</td>
<td>0.139</td>
<td>0.393</td>
<td>0.225</td>
<td>-2.32</td>
<td>0.094</td>
<td>6.307</td>
</tr>
</tbody>
</table>
coelastic behaviour are given in the appendix. There is eqn (13) in the appendix which provides a satisfactory fit of the viscosity data, \( \eta(\dot{\gamma}) \) vs. \( \dot{\gamma} \), for obtaining \( a_0, b_0, \) and \( \eta_0 \) for PAA solutions in this work (Table 1) as can be seen from Figure 2. The parameters in eqn (14) such as \( \lambda_0, c \) and \( d \) can be calculated by using \( \eta_0 \) from eqn (13) and curve fitting of \( \eta'(\omega) \) vs. \( \omega \) using eqns (14) and (15).

The resulting values obtained by nonlinear regression of the modified upper convected Maxwell model [16] are given in Table 1. The rheograms of test fluids are shown in Figure 3.

RESULTS AND DISCUSSION

Eleven radial positions from the impeller tip to the vessel wall were chosen in order to measure the mean velocity components in mixing of PAA solutions. Local mean velocity and velocity fluctuations at these points have been measured for different rotational speeds of Rushton turbine impeller and various concentrations of PAA solutions.

Tangential Velocity

Figure 4 shows an example of the local tangential velocities at different radial positions obtained through LDA measurements. These velocities are normalized by the blade tip speed. Dimensionless tangential velocity distributions for different concentrations of PAA solutions, a, b, c and d of Table 1 at various impeller speeds show that the velocity profiles for different impeller speeds have the same exponential decay and nearly are independent of impeller rotational speed. These results have important design implications with respect to scale-up rules and are in agreement with the literature [11, 17].

It should be noted that this independency does not apply to pitched blade turbine because of the existence of different flow patterns [12]. Figures 4 and 5 show that the steep reduction in dimensionless tangential velocities of the Rushton turbine impeller at different impeller speeds and concentrations occurs in the range of \( 0.4 < r/R < 0.6 \). Also, in contrary to the findings of Stoots and Calabrese [18] for Newtonian liquids, these figures show that the dimensionless mean tangential velocities cannot exceed the impeller speed, i.e., \( \bar{v}_t/\nu_{\text{tip}} < 1.0 \). A comparison between \( \bar{v}_t/\nu_{\text{tip}} \) at an impeller speed of about 23 rpm for different PAA solutions in this work is shown in Figure 5. It is deducted from the data of the given radial position that \( \bar{v}_t/\nu_{\text{tip}} \) ratio decreases with increasing fluid concentration.
The mean tangential velocity profiles for different concentrations of PAA solutions and impeller speeds in the lower transition region (i.e., $35 < \text{Re}' < 1800$) satisfy the following equation:

$$\frac{v_t}{v_{\text{tip}}} = m + n \exp\left(-\frac{r}{RL}\right)$$  \hspace{1cm} (5)

For the first time, an equation similar to eqn (2) is suggested for $\frac{v_t}{v_{\text{tip}}}$ as:

$$\frac{v_t}{v_{\text{tip}}} = 0.85 \xi_t \left(\frac{r}{R}\right)^{-7/6}$$  \hspace{1cm} (6)

where $\xi_t$ can be obtained from curve fitting of $\frac{v_t}{v_{\text{tip}}}$ data versus $r/R$ for different PAA solutions and different impeller speeds such as done in Figure 4. The variation of $\xi_t$ versus Reynolds number is shown in Figure 6. This figure shows that $\xi_t$ is proportional to $\text{Re}'$ for $\text{Re}' < 90$ in the case of 1350 ppm PAA solution. Also, for low concentration of PAA solution (i.e., less than 1000 ppm), it is nearly equal to 0.19 in the range of $90 < \text{Re}' < 1800$. On the other hand, deviations from turbulent flow (deviations from eqn (1)) for different PAA solutions are nearly the same in the latter range of $\text{Re}'$. In addition, the viscoelastics of PAA solutions in the range of higher Reynolds numbers, $90 < \text{Re}' < 1800$, have little effect on $\xi_t$.

The mean values of the coefficients of eqns (5) and (6) i.e., $m$, $n$, $L$, and $\xi_t$ with their standard deviations (SD) [19] are given in Table 2. Further, this table contains the mean relative deviations, i.e., $\bar{E}$ of the these equations obtained through using local relative error, i.e., $\varepsilon_i$. Besides, Table 2 shows a greater $\bar{E}$ for eqn (6) than eqn (5). Figure 7 gives a comparison between eqns (5) and (6) revealing that eqn (5) correlates $\frac{v_t}{v_{\text{tip}}}$ data better. On the other hand, if mean values of $m$, $n$, $L$, and $\xi_t$ for PAA solutions are used in eqns (5) and (6), it will be found that the mean relative deviations are 11.7 and 16 percent, respectively. Therefore, it can be concluded that eqns (5) and (6) are the more preferred correlations for $\frac{v_t}{v_{\text{tip}}}$ data in Figures 4 and 5.

### Radial Velocity

A similar approach correlates the radial component. Figure 8 shows the dimensionless radial velocity,

<table>
<thead>
<tr>
<th>Coef. Fluid</th>
<th>Coef.</th>
<th>SD</th>
<th>Coef.</th>
<th>SD</th>
<th>Coef.</th>
<th>SD</th>
<th>Coef.</th>
<th>SD</th>
<th>Coef.</th>
<th>SD</th>
<th>Coef.</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$m$</td>
<td></td>
<td>$n$</td>
<td></td>
<td>$L$</td>
<td></td>
<td>$\xi_t$</td>
<td></td>
<td>$\bar{E}$ of eqn (5)</td>
<td></td>
<td>$\bar{E}$ of eqn (6)</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>0.088</td>
<td>0.018</td>
<td>6.5</td>
<td>2.223</td>
<td>0.162</td>
<td>0.018</td>
<td>0.187</td>
<td>0.014</td>
<td>0.131</td>
<td>0.171</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>0.081</td>
<td>0.015</td>
<td>8.02</td>
<td>1.874</td>
<td>0.173</td>
<td>0.015</td>
<td>0.184</td>
<td>0.012</td>
<td>0.109</td>
<td>0.156</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>0.05</td>
<td>0.022</td>
<td>4.303</td>
<td>0.9</td>
<td>0.211</td>
<td>0.023</td>
<td>0.2</td>
<td>0.012</td>
<td>0.111</td>
<td>0.153</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>0.073</td>
<td>0.018</td>
<td>6.274</td>
<td>1.66</td>
<td>0.182</td>
<td>0.019</td>
<td>0.190</td>
<td>0.0126</td>
<td>0.117</td>
<td>0.16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 5.** Tangential velocities at discharge plane for different concentrations of PAA solutions.

**Figure 6.** Variation of $\xi_t$ as a function of $\text{Re}'$ number for various concentrations of PAA solutions.

**Table 2.** The mean values of coefficients in eqns (5) and (6) with their standard deviations (SD) and mean relative deviations ($\bar{E}$) for different PAA solutions.
\( \frac{\nu_r}{\nu_{tip}} \), for 900 ppm PAA solution at different impeller speeds corresponding to the lower transition region, i.e., \( 35 < \text{Re}' < 1800 \). This figure shows that impeller speed affects \( \frac{\nu_r}{\nu_{tip}} \) and in each case \( \frac{\nu_r}{\nu_{tip}} \) is much less than what eqn (1) suggests and becomes smaller when \( \text{Re}' \) decreases. In comparison with \( \frac{\nu_t}{\nu_{tip}} \), the \( \frac{\nu_r}{\nu_{tip}} \) data have a more complex behaviour and its value is smaller and it is also in agreement with the results of Kai et al. [20].

As can be seen from Figure 9, the profiles of \( \frac{\nu_r}{\nu_{tip}} \) data, like \( \frac{\nu_t}{\nu_{tip}} \) data, cannot exceed the blade tip speeds. Also, like \( \frac{\nu_t}{\nu_{tip}} \) data, the latter result is not similar to the findings of Stoots and Calabrese [18].

Figures 8 and 9 further show that the region in which the profile of \( \frac{\nu_r}{\nu_{tip}} \) reaches a maximum, is in the range of \( 0.4 < r/R < 0.6 \). This behaviour may be due to elastic effects in the vicinity of impeller blade such as the Weissenberg effect [21]. The distributions of \( \frac{\nu_r}{\nu_{tip}} \) data of 900 and 1350 ppm PAA solutions in Figures 8 and 9 deviate from straight line at locations much further than the impeller tip, i.e., \( r/R > 0.7 \) [22]. The velocity approaches zero and this causes the formation of a pseudo-cavern [11, 23]. This phenomenon is a cavern that does not have well-defined boundaries between the stagnant and well-mixed regions. It can be due to the low shear regions formed away from the impeller tip. These results support previous observations by Metzner and Taylor [24] that it is possible to have both laminar and turbulent regimes in an agitated vessel at the same time.

In order to determine the \( \frac{\nu_r}{\nu_{tip}} \) profiles, \( \xi_r \) is introduced in eqn (1) as follows:

\[
\frac{\nu_r}{\nu_{tip}} = 0.85 \xi_r (r/R)^{-7/6} \quad (7)
\]

Also, the following equation well correlates with
The variations of $\xi_r$ versus $Re'$ numbers for different PAA solutions are shown in Figure 10. This figure shows that $Re' \propto \xi_r$ in the range of $Re'<145$ for 1350 ppm PAA, and reaches a maximum at about $Re'=1160$ for 500 ppm PAA solution. The maximum in Figure 10 for PAA solutions do not exist in Newtonian [12] and inelastic non-Newtonian systems [11]. This figure shows that $\xi_r$ (in eqn (7)) decreases with increasing elasticity of PAA solutions and decreasing of $Re'$ number. A comparison between $\xi_r$ in this work with that for Newtonian and inelastic non-Newtonian fluids in the literature [11,12] is shown in Figure 11. The results in this figure show that $\xi_r$ for PAA solutions is smaller than those for Newtonian and inelastic non-Newtonian liquids. On the other hand, deviations from turbulent flow (eqn (1)) for different PAA solutions are very distinct.

Values of $\xi_r$, $a_1$, and $b_1$ in eqns (7) and (8) can be obtained from curve fitting of $\nu_r/\nu_{tip}$ data as in Figures 8 and 9. The mean values of these coefficients for different PAA solutions are tabulated in Table 3. This table shows that eqn (7) has larger $\bar{E}$ with respect to eqn (8). Also, eqn (7) fails to correlate the radial velocity data at some speeds but eqn (8) can well correlate all of the radial velocity data of this work. Figure 12 shows a comparison between eqns (7) and (8) with optimum values of $\xi_r$, $a_1$, and $b_1$. The mean relative deviations of $\nu_r/\nu_{tip}$ data (eqns (7) and (8)) of PAA solutions (Table 1) are 14.5 and 10.9 %, respectively. It can be concluded that the use of $\xi_r$ is a suitable approach for correlating $\nu_r/\nu_{tip}$ data in Figures 8 and 9.

**CONCLUSION**

LDA velocity measurements with polyacrylamide (PAA) solutions for a typical Rushton turbine have produced results in the lower transition region, i.e.,
35 < \text{Re}^\prime < 1800 as:

1. The dimensionless mean (radial and tangential) velocity as a function of dimensionless radial position are nearly independent of impeller speed.

2. Correlations for mean radial velocities of Newtonian and inelastic non-Newtonian fluids do not apply to the case of viscoelastic polymeric liquids.

3. The mean tangential and radial velocity profiles at discharge plane of Rushton turbine impeller were reasonably correlated by introducing the correlation parameters, $\xi_t$ (for the first time) and $\xi_r$, in Vander Molen and Van Maanen equation [8], i.e., eqn (1).

4. $\xi_t$ in eqn (6) has nearly constant value of 0.19 for dilute concentration of PAA solutions, e.g., lower than 1000 ppm, but for higher concentrations of PAA solution, i.e., 1350 ppm PAA solution, it decreases sharply.

5. $\xi_r$ in eqn (7) decreases as elasticity of PAA solutions increases and its values for PAA solutions are smaller than those for Newtonian and inelastic non-Newtonian liquids.

**NOMENCLATURE**

- $a_0, b_0, a_1, b_1, c, d, L, m, n$: Constants
- $d$: Stretch tensor ($d = 0.5(V_v + (V_v)^T)$)
- $D$: Impeller diameter, m
- $E_l$: Elasticity number defined by eqn (11)
- $E$: Mean relative deviation
- $N$: Rotational speed, rev/s, rpm
- $N_1$: First normal stress difference, Pa
- $r$: Radial coordinate, m
- $R$: Radius of agitated vessel, m
- $\text{Re}$: Newtonian Reynolds number = $\rho ND^2/\mu$, dimensionless
- $\text{Re}_a$: Apparent Reynolds number = $\rho ND^2/\mu_a$, dimensionless
- $\text{Re}^\prime$: Zero shear rate Reynolds number = $\rho ND^2/\eta_0$, dimensionless
- $T$: Agitated vessel diameter, m
- $\text{SD}$: Standard deviation
- $v_r$: Radial velocity, m/s
- $v_t$: Tangential velocity, m/s
- $v_{tip}$: Impeller blade tip speed = $\pi DN$, m/s
- $Wi$: Weissenberg number defined by eqn (9)

**Greek Letters**

- $\gamma_0$: Amplitude of complex shear rate, 1/s
- $\dot{\gamma}_T$: Impeller tip shear rate, 1/s
- $\varepsilon_i$: Local relative error = $((\nu_t/v_{tip})_{\text{exp}} - (\nu_t/v_{tip})_{\text{calc}})/(\nu_t/v_{tip})_{\text{exp}}$
- $\eta_0$: Zero shear rate viscosity, kg/ms
- $\eta(\gamma)$: Steady state viscosity, kg/ms, eqn (13)
- $\eta'(\omega)$: Oscillatory viscosity, kg/ms, eqn (14)
- $\lambda_0$: Zero shear rate Maxwell relaxation time, s
- $\mu_a$: Apparent viscosity, kg/ms
- $\mu$: Newtonian viscosity, kg/ms
- $\tau_{12}$: Shear stress, Pa
- $\rho$: Density, kg/m$^3$
- $\xi$: Van der Molen and Van Maanen s correlating constant, eqn (2)
- $\xi_r$: Van der Molen and Van Maanen s correlating constant for radial velocity
- $\xi_t$: Van der Molen and Van Maanen s correlating constant for tangential velocity
- $\psi_{12}$: Primary normal stress difference coefficient, Pa.s$^2$
- $\phi$: Oscillation amplitude, degree
- $\omega$: Angular frequency of oscillatory shear flow, 1/s
- $I_{1d}$: Second invariant of the strain tensor, $I_{1d} = 0.5(\nabla_v \nabla_v)$

**Superscript**

- $-\cdot$: Time or space value
- $o$: Amplitude of a complex quantity
- $\dagger$: Transpose of a second order tensor

**Subscripts**

- calc: Calculated values by proposed correlations
- exp: Experimental data
- $r, t$: Radial and tangential components

**APPENDIX**

**Dimensionless Groups**

A dimensionless number that characterizes viscoelastic properties is the Weissenberg number defined by:

$$Wi = N_1/\tau_{12} = \lambda_0 \dot{\gamma}_T$$  \hspace{1cm} (9)

The Weissenberg number compares the magnitude of normal stresses to shear stresses. Other definitions except the above mentioned definitions, can be found in the literature [21]. The first normal stress difference,
N₁, is given by the following equation:

\[ N₁ = \tau_{11} - \tau_{22} = \psi_{12} (\dot{\gamma})^2 \]  

(10)

It has been shown that the key group for reversal flow pattern in mixing of viscoelastic fluids is the elasticity number \([s²]\) defined as follows:

\[ El = \frac{W_i}{Re'_{}} = \frac{\psi_{12}}{\rho D^2} \]  

(11)

where:

\[ Re'_{} = \frac{\rho ND^2}{\eta_0} \]  

(12)

According to the modified upper convected Maxwell model \([16]\) the following functional form for \(\eta(\dot{\gamma})\) and \(\eta'(\omega)\) have been chosen in this work:

\[ \eta(\dot{\gamma}) = \eta_0 / [1 + a_0 I_{II_d}^{b_0}] \]  

(13)

where \(I_{II_d}\) is the second invariant of the rate of strain tensor in steady shear flow and taken, in this work, as equal to \(I_{II_0} = 0.25\dot{\gamma}^2\)

\[ \eta'(\omega) = \eta_0 (1 + c I_{II_d}^d) / [(1 + c I_{II_d}^d)^2 + \lambda_0^2 \omega^2] \]  

(14)

Also, the second invariant of rate of strain tensor for oscillatory shear flow is defined as:

\[ I_{II_d} = \frac{1}{4} \dot{\gamma}^2 = [(1/12) M \phi \omega]^2/4 = 4.7597 \times 10^{-4} \omega^2 \]  

(15)

The constant \(M = 3\) is a shear rate factor, depending on the sensor system of CV100. Rheometer and oscillation amplitude, \(\phi\), is taken as \(10^9 \times \pi/180\) (rad), in this work.

REFERENCES


